

# NUMERICAL STUDY OF A NEW BUNCH LENGTH MONITOR UTILIZING A DETECTION OF ELECTROMAGNETIC FIELDS IN MILLIMETER-WAVE REGION

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## Abstract

A new nondestructive bunch-length monitor has been numerically investigated in high-energy electron/positron linear accelerators. The monitor detects electromagnetic radiation emission generated in wave zone through a gap of a vacuum pipe when a relativistic bunched beam passes across the pipe gap. The frequency spectrum of the radiation emission spreads over microwave to a millimeter-wave region for the bunched beam with a bunch length of a picosecond region. The frequency spectrum strongly depends on the bunch length if the geometrical structure of the pipe gap is suitably fixed. The detection principle of the new bunch-length monitor and some numerical analysis results applied to a single-bunch electron beam at the KEKB injector linac are described in this report.

## INTRODUCTION

A bunch-length measurement of a bunched beam is one of important beam diagnostics in linear accelerators. There are several methods to measure the bunch length [1]. One is a standard method with a streak camera. It measures a pulse width of electromagnetic radiations in time domain, such as Cherenkov radiation and optical transition radiation generated through electromagnetic interactions between materials and the bunched beam. This method comes in useful for the bunch-length measurement in a picosecond region. Recently, a fast streak camera can make the bunch-length measurement possible with a time resolution of sub-picoseconds [2].

These are well-known methods to measure the bunch length in a pico (or sub-pico) second region, and however, they have several drawbacks because they are basically based on destructive diagnostics, and they need expensive instrumentations with complicated systems.

Here, we propose an alternative new method to measure the bunch length in frequency domain by nondestructively detecting electromagnetic radiation emissions. They are shaken off the bunched beam by diffraction (or scattering) at a pipe gap of a beam line. This method requires only a pipe gap normally vacuum-sealed with a ceramic. It does not require any other devices in the beam line.

There are many pipe gaps vacuum-sealed with a conventional ceramic along the beam line of the KEKB injector linac in order to install other beam diagnostic devices. Such pipe gaps are available for the bunch-length measurement based on this new method at any locations of the entire beam line

In this report, the basic principle on this new method

are presented. The characteristics of the electromagnetic radiation emitted from the pipe gap are also given, and some numerical results analyzed for a single-bunch electron beam at the KEKB injector linac are summarized.

## CHARACTERISTICS OF THE RADIATION EMISSION

When a charged particle with a relativistic energy passes through a vacuum pipe in linear accelerators, image charges with the inverse sign are induced on the inner surface of the pipe, and they also flow simultaneously through it. Electromagnetic fields (or self-fields) induced between the charged particle and the image charges are relativistically boosted in the longitudinal direction. When such pancake-like self-fields pass across the pipe gap, a part of the self-fields are emitted out of the pipe gap in wave zone because the self-fields must be met with the electromagnetic boundary conditions under which the charged particle is surrounded with both the pipe gap and the metal pipe.

The characteristics of the electromagnetic radiation emission from the charged particle are determined by depending on the geometrical structure of the pipe gap (see fig. 1(a)) if the charge and the energy of the charged particle are constant.

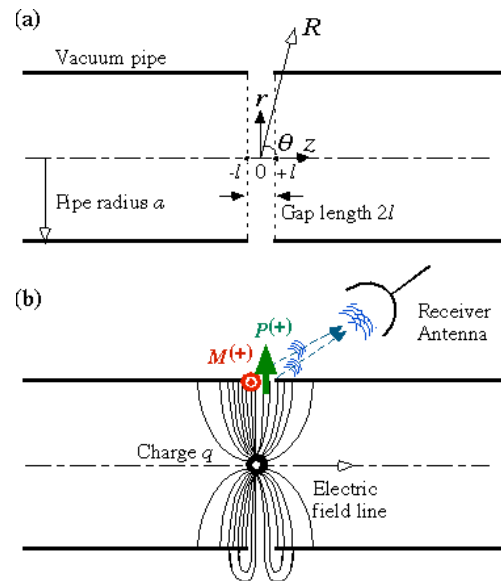


Figure 1: (a) Geometrical structure of the vacuum pipe and the pipe gap and (b) a schematic drawing of the electromagnetic radiation emission through the pipe gap.

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On the other hand, when a bunched beam goes across the pipe gap, the characteristics of the electromagnetic radiation emission may be modified depending not only on the geometrical structure of the pipe gap, but also the geometrical charge distributions of the bunched beam.

Based on the Bethe's diffraction theory by small holes [3], a flow of electromagnetic radiations through such small holes may induce electric and magnetic dipole moments on the surface of the small holes. The vector directions of these dipoles correspond to those of the field vectors induced on the surface of the closed holes. The time oscillation of the induced dipoles can emit the electromagnetic radiations in wave zone out of the small holes. A schematic drawing on the radiation emission based on the dipole oscillations is shown in fig. 1(b). One of the characteristic features of the radiation emission is that the emission intensity becomes a maximum (minimum) in the direction of the beam axis ( $r$  axis).

On the other hand, an emission of rf waves out of a finite open waveguide with an rf oscillator at one end was exactly investigated by Weinstein [4]. Based on the Weinstein's theory, when the relativistic bunch passes across the pipe gap, a large part of the self-fields transmits across the pipe gap, and however, a part of the self-fields reflects at both open ends of the pipe gap to be met with the electromagnetic boundary conditions. Thus, the multiple reflections of the self-fields occur between both the open ends, and a part of the interfered self-fields is emitted into free space out of the pipe gap due to diffraction (or scattering) (see fig. 1(b)).

It is generally very difficult to exactly solve these electromagnetic radiation emission problems. Palumbo [5] and Kheifets [6] give an analytical formula on an energy loss of the charged particle generated at such a pipe gap by approximately working out these problems based on the Weinstein's theory. The energy loss (or radiation power) per a solid angle  $d\Omega$  is given by

$$\frac{dW(\theta)}{d\Omega} = \frac{\alpha\hbar\beta(q/e)^2 \sin^2\theta \cdot J_0^2(ka \sin\theta)}{\pi(1-\beta \cos\theta)^2 I_0^2(ka/\beta\gamma)} \times \left| \frac{L_-(\omega/v)\sqrt{1-\beta}}{L_-(k \cos\theta)\sqrt{1-\cos\theta}} e^{-jkl(1-\beta \cos\theta)/\beta} - j \frac{L_+(\omega/v)\sqrt{1+\beta}}{L_+(k \cos\theta)\sqrt{1+\cos\theta}} e^{jkl(1-\beta \cos\theta)/\beta} \right|^2 \quad (1)$$

Here,  $\alpha$  the fine structure constant,  $\hbar$  the reduced plank constant,  $q$  the particle charge,  $e$  the electron charge magnitude,  $\beta=v/c$ ,  $v$  the velocity of the charged particle,  $\theta$  the spatial emission angle,  $\omega$  the angular frequency of the electromagnetic radiation,  $k=\omega/c$ ,  $c$  the velocity of light,  $\gamma$  the relativistic Lorentz factor,  $J_0$  the Bessel function of the first kind,  $I_0$  the modified Bessel function of the first kind. The other parameters are indicated in fig. 1(a). It should be noted that the higher-order multiple-reflection terms are neglected in Eq. (1) due to the smaller contribution. The right-hand side of Eq. (1) shows that the radiation spectrum mainly comprises two terms for which the first (second) term makes a contribution from the radiation emission generated at the left (right)-side end of the pipe gap (see fig. 1(b)). The functions  $L_{\pm}$  are derived

on the basis of the Wiener-Hopf factorization method [4] by

$$L_+(\alpha) = j\Gamma_+(\alpha)\sqrt{\alpha+k} \quad (2)$$

$$L_-(\alpha) = -j\sqrt{k-\alpha}/\Gamma_-(\alpha). \quad (3)$$

The functions  $\Gamma_{\pm}$  are specified [7] by

$$\Gamma_{\pm}(\alpha) = [2\Sigma I_0(\Sigma)K_0(\Sigma)]^{\pm 1/2} \exp \left\{ \frac{\alpha a}{j\pi} \int_0^{ka} dt \frac{\ln[\pi\Sigma_1 J_0(\Sigma_1)H_0^{(1)}(\Sigma_1)]}{\alpha^2 a^2 - t^2} + j \frac{\alpha a}{\pi} PV \int_{ka}^{\infty} dt \frac{\ln[2\Sigma_2 I_0(\Sigma_2)K_0(\Sigma_2)]}{t^2 - \alpha^2 a^2} \right\}. \quad (4)$$

Here,  $H_0^{(1)}$  is the Hankel function of the first kind,  $K_0$  is the modified Bessel function of the second kind, and the other parameters are given by

$$\Sigma^2 = a^2(\alpha^2 - k^2), \Sigma_1^2 = k^2 a^2 - t^2, \Sigma_2^2 = t^2 - k^2 a^2. \quad (5)$$

It should be noted that Eq. (1) gives the relativistic radiation emission formula in frequency domain.

## NUMERICAL STUDIES OF THE RADIATION EMISSION

In general, the radiation emission spectra from the bunched beam may be derived from the convolution of Eq. (1) with the spatial charge distributions. It is easy to numerically solve the integrations rather than to calculate them analytically. Assuming the longitudinal charge distribution of the bunched beam being expressed by a Gaussian function, several number of macro particles are arranged discretely at equal distance along the longitudinal direction.

Therefore, the radiation emission spectra can be numerically investigated by summing the imaginary amplitude contributions from all the macro particles following Eq. (1). Here, the number of macro particles was assumed to be 6 arranged within the total width of  $\pm 3\sigma_l$  ( $\sigma_l$ , one-sigma bunch length) of the longitudinal charge distribution. The gap length and the pipe diameter were fixed to be  $2l=12\text{mm}$  and  $2a=60\text{mm}\phi$ , respectively.

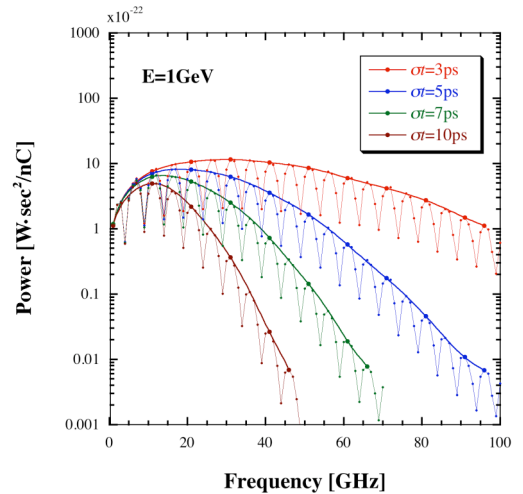


Figure 2: Radiation emission spectra as functions of the frequency and the bunch length.

The results at the beam energy of 1 GeV are shown in fig. 2. Here, for the sake of simplicity, the rf loss at a ceramic seal of the pipe gap and the energy spread of the single-bunch beam were neglected. The radiation emission angle was fixed to be  $\theta=\pi/2$ . As shown in fig. 2, the radiation emission powers are larger at the millimeter-wave region over the microwave region for the shorter bunch length in a picosecond region. The spectra also indicate periodical dip structures due to diffraction (or scattering). The frequency and the depth at the dip location are determined by the geometrical structure of the pipe gap. The solid curves in the figure are envelopes without the dip structures. The result (blue line,  $\sigma_t=5$  ps) corresponds to that for the KEKB injection beam since the bunch length is  $\sim 10$  ps in FWHM. The difference of each emission spectrum is so remarkable in millimeter-wave region ( $>20$  GHz) even for the bunch length difference of a few picoseconds.

The radiation emission spectra based on the envelopes depending on the beam energy are shown in fig. 3 while the bunch length is fixed to be  $\sigma_t=5$  ps.

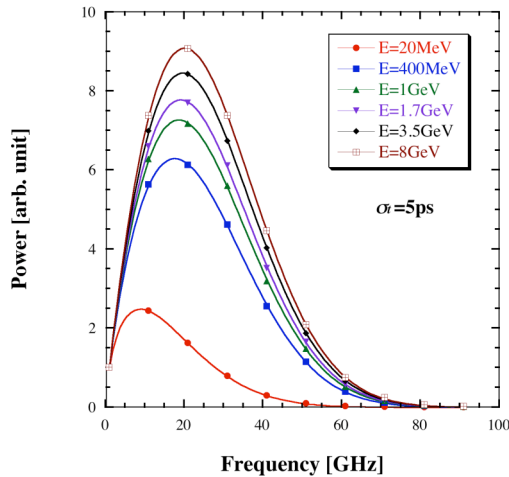


Figure 3: Radiation emission spectra as functions of the frequency and the beam energy.

It should be noted that the radiation power spectra are normalized with each power at the frequency of 1 GHz. It is understood that the energy dependence of the critical frequency is not very large at highly relativistic energies while the critical frequency giving the maximal power of each distribution increases with the increase of the beam energy. This may come from the relativistic Lorentz boost of the self-fields.

Figure 4 shows the variations in the normalized radiation power spectra as functions of the critical frequency, the bunch length, and the beam energy. The results indicate that the radiation power at the critical frequency increases with the increase of the beam energy, and the critical frequency largely shifts towards the higher frequency region for the shorter bunch length. It is of great benefit to this method for the shorter bunch-length measurement. The results also give the mapping curves as functions of the bunch length and the beam energy.

Assuming that the frequency resolution in the critical frequency measurement is attainable within  $\delta f \sim 1$  GHz over the millimetre-wave region ( $f=10$ -30GHz), the resolution of  $\delta\sigma_t \sim 1$  ps may be attainable enough although it is not easy to measure the precise beam energy with this method at highly relativistic energies.

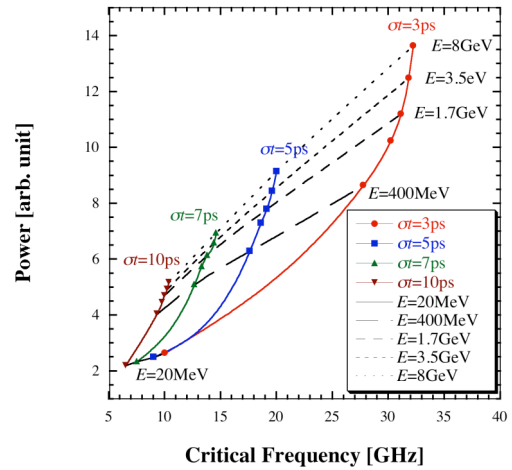


Figure 4: Relations (mapping curves) of the critical frequency to the radiation power as functions of the bunch length and the beam energy.

An example of the detection system is simply presented here. It comprises a receiver antenna, a tunable band-pass filter with a narrow bandwidth, and a peak-power meter. The millimetre waves are directly detected with the receiver antenna installed near the pipe gap. The power spectra are measured through the band-pass filter by sweeping the centre frequency over the frequency range of 5-40 GHz. The peak power obtained is  $\sim 54$  dBm/nC, if the bandwidth of the detectable frequency is  $\Delta f \sim 1$  MHz. It seems to be possible by using a commercially available peak-power meter at millimetre-wave region.

## SUMMARY

A new nondestructive bunch-length monitor has been numerically investigated for a single-bunch beam of the KEKB injector linac. The numerical results show that the bunch length with a resolution of  $\delta\sigma_t \sim 1$  ps may be measured based on this new method with a suitable detection system.

## REFERENCES

- [1] A.W.Chao and M.Tigner (eds.), *Handbook of Accelerator Physics and Engineering* (World Scientific, Singapore, 1999), p.547.
- [2] <http://jp.hamamatsu.com/en/index.html>.
- [3] H.A.Bethe, Phys. Rev. 66, 163 (1944).
- [4] L.A.Weinstein, *Theory of Diffraction and the Factorization Method* (The Golem Press, Colorado, 1969).
- [5] L.Palumbo, CERN LEP-TH/84-4.
- [6] S.Kheifets, et al., CERN LEP-TH/85-23.
- [7] B.W.Zotter and S.A.Kheifets, *Impedance and Wakes in High-Energy Particle Accelerators* (World Scientific, Singapore, 1998), p.267.