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Transient Beamloading Analysis for Linac's RF Control

M. A. Chernogubovsky

Japan Atomic Energy Research Institute (JAERI)
Tokai-mura, Naka-gun, Ibaraki 319-11, Japan

ABSTRACT
The beam excitation properties at transient beamloading in linac are analyzed. Effective RF control design and the control signal synthesis are considered at the directional selective coupling principle application. The results on RF system characteristics determination for the accelerating structure precise tuning, for RF matching, and the initial results on the control signal optimization are presented.

1 Introduction

The high-duty-factor linac's main specifications lead to a need for excellent control of the accelerating fields to obtain the high-quality beam characteristics and to minimize the beam loss possibility and the resulting activation [1]. The well-developed [2,3] approximate method, assuming a slow variation of amplitude and phase for the single harmonic field, is inadequate for the rigorous analysis. Beamloaded resonator equivalent circuit contains nonlinear elements [4] even at [2,3] approximation and compete analysis of the external properties is required also.

2 RF general characteristics

The resonant accelerator design usually employs single mode principle with the single-harmonic time dependence of the electric field,

$$e_1(t) = A\cos(\omega_0 t + \varphi)\,. \qquad (1)$$

Particles dynamics analysis determines tolerable deviations $\Delta A, \Delta\omega_0, \Delta\varphi$ at this approximation with addition of the gradient (self-Coulomb) field. Thus, RF control system is to be designed for this operating condition implementation.

At eigenfunctions expansion the resonator ν-mode (spatial distribution $\{\vec{E}_\nu(\vec{R}), \vec{H}_\nu(\vec{R})\}$, eigenfrequency ω_ν, selfquality factor Q_ν, norm W_ν) time dependencies $e_\nu(t), h_\nu(t)$ are defined by

$$\begin{cases} (1+\frac{1}{Q_\nu})h_\nu'' + \frac{\omega_\nu h_\nu'}{Q_\nu} + \omega_\nu^2 h_\nu = \frac{-\omega_\nu(In_L + In_B)}{W_\nu}, \\ In_L(t) = \int\limits_V \left(\vec{E}_\nu(\vec{R}), \vec{j}_L(\vec{R},t)\right) dV\,, \\ In_B(t) = \int\limits_V \left(\vec{E}_\nu(\vec{R}), \vec{j}_B(\vec{R},t)\right) dv\,, \\ e_\nu = \frac{1}{\omega_\nu}(1+\frac{1}{Q_\nu})h_\nu' + \frac{1}{Q_\nu}h_\nu\,; \end{cases} \qquad (2)$$

where $\vec{j}_B$ and $\vec{j}_L$ are the beam and the coupling elements current densities. The use of grid-control device for RF source provides the relatively wide band control signal, and directional selective coupling [5] puts away the undesirable modes problem. The linear RF system (m identical loops) is presented by equivalent current source I_s with output impedance ρ at each loop reference plane, set (2) self-consistent solution for simplified coupling device is

$$h_1'' + \frac{\omega_r}{Q_L}h_1' + \omega_r^2 h_1 = S_s(t) + V(t)\,, \quad e_1 = \frac{h_1'}{\omega_r}\,; \qquad (3)$$

loaded quality factor of the operating mode Q_L is

$$Q_L = \frac{1}{\frac{1}{Q_r} + \frac{m\omega_r(\mu K_l)^2}{\rho W_1}}\,, \quad K_l = \int\limits_{S_l} (\vec{H}_1, d\vec{S})\,; \qquad (4)$$

S_l, ω_r, Q_r – loop space, resonant frequency and unloaded quality factor; the beam excitation and source signals $V(t) = -\omega_r \int\limits_V (\vec{E}_1(\vec{R}), \vec{j}_B(\vec{R},t))dv/W_1$, $S_s(t) = I_S(t)\omega_r^2 \mu m K_l/W_1$. Consider the beam with as much as desired but finite duration τ_{in}, which leading front enters the resonator at $t=0$, the source signal $S_s(t) = S_-(t) + S(t)$; $S_-(t)$ was started before $t=0$ and sets up the required field (1), whereas $S(t)$ starts at $t=0$. The form (3) linearity yields the condition for $S(t)$ synthesis: the solution h of

$$h'' + \frac{\omega_r}{Q_L}h' + \omega_r^2 h = S(t) + V(t) \qquad (5)$$

($h''(0) = h'(0) = h(0) = 0$) must be zero[1] for $t>0$ and the control system must hold up $V(t)+S(t)+S_-(t)|_{t>0}$ variations in required limits. To define Fourier $\mathbf{F}\{i\omega\}$ spectrum $V(\omega)$ consider the energy variation $\mathcal{E}_a$ of

[1] Trivial solution $S(t) = -V(t)$ may be unrealizable since the limited bandwidth of the source, in addition it is far from the best in the meaning of the required RF energy.

the beam particle a with charge $\overline{e}$, which moves on $\vec{r_a}(t)$ trajectory with $\vec{v_a}(t)$ velocity in a mode field $\vec{E} = e_\nu(t)\cdot\vec{E_\nu}(\vec{R})$, [6]: $\frac{d\mathcal{E}_a}{dt} = \overline{e}\left(\vec{v_a}(t), \vec{E}\right) = e_\nu(t)\cdot\overline{e}\left(\vec{v_a}(t), \vec{E_\nu}(\vec{r_a}(t))\right)$. The (2) integral $In_B(t) = \sum\limits_a \begin{bmatrix} \overline{e}\left(\vec{v_a}(t), \vec{E_\nu}(\vec{r_a}(t))\right) & a \text{ in the resonator} \\ 0 & \text{otherwise} \end{bmatrix}$, so that $e_\nu(t)\cdot In_B(t) = \sum\limits_a \frac{d\mathcal{E}_a}{dt}$, where the extension of $\mathcal{E}_a(t)$ definition on the constant value eliminates the bivariant condition[2]. Thus, $V(t)$ satisfies the equation

$$A\cos(\omega_0 t + \varphi)\cdot V(t) = \frac{-\omega_r}{W_1}\sum_a \frac{d\mathcal{E}_a}{dt} = F(t). \quad (6)$$

As far as $\omega_0 \approx \omega_r$, and the Eq.(5) left-hand side operator has the narrow bandwidth, the desirable solution is obtained at $V(\omega)$ compensation by $S(\omega)$ only in $[\omega_L, \omega_h]$ band, Fig. 1a. The ω_L, ω_h values are determined for concrete control system under admissible $\Delta A, \Delta\omega_0, \Delta\varphi$.

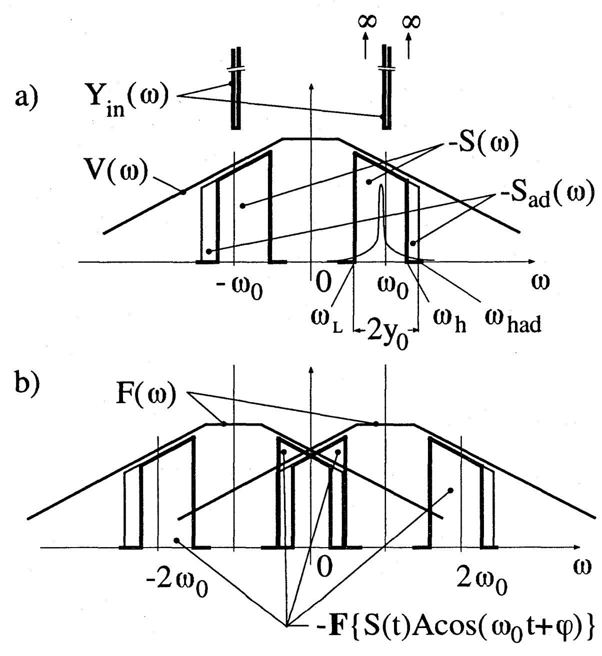

Fig. 1. Spectrums transformations according to Eq.(6) and input conductance $Y_{in}(\omega)$.

At the required solution of Eq.(5) with (1) form of $e_1(t)$ the input conductance in the reference plane is

$$Y_{in}(\omega) = \frac{W_1}{m\,(\omega_r \mu K_l)^2}\cdot\left(\frac{\omega_r(1+i\xi)}{Q_r} - \frac{V(\omega)}{A\pi\omega_r[e^{i\varphi}\delta(\omega-\omega_0) + e^{-i\varphi}\delta(\omega+\omega_0)]}\right), \quad (7)$$

where $\xi = \frac{\omega^2 - \omega_r^2}{\omega\omega_r}Q_r$. Finite values are only at $\omega = \pm\omega_0$, Fig. 1a, since the I_s source spectrum is finite, while the field contains single harmonic and the loop voltage is nonzero at ω_0 only. So, at $\omega \neq \omega_0$ all the incident power will be reflected, this property provides support signals for the control system. The value on ω_0 is determined by selecting the component $V_0\pi[e^{iv_0}\delta(\omega-\omega_0) + e^{-iv_0}\delta(\omega+\omega_0)]$ [3] in the beam excitation signal, so the resonator must be tuned in accordance with

$$\omega_0^2 - \omega_r^2 = \frac{\omega_0}{\omega_r}\cdot\frac{V_0}{A}\sin\psi, \quad (8)$$

$\psi = v_0 - \varphi$, for $\Im\{Y_{in}(\omega_0)\} = 0$ resonance condition; therewith RF source matching is realized at

$$\frac{1}{\rho} = \Re\{Y_{in}(\omega_0)\} = \frac{W_1\left(\frac{\omega_r}{Q_r} - \frac{V_0\cos\psi}{A\omega_r}\right)}{m\,(\omega_r\mu K_l)^2}.$$

The same $Y_{in}(\omega_0)$ value is obtained for the resonator without the beam ($V_0 = 0$ in Eq.(7)) but with new resonant frequency ω_{eq} and new unloaded quality factor Q_{eq}:

$$\omega_{eq} = \sqrt{\omega_r^2 + \frac{\omega_0 V_0 \sin\psi}{\omega_r A}},\quad Q_{eq} = \frac{\omega_{eq}}{\frac{\omega_r}{Q_r} - \frac{V_0\cos\psi}{A\omega_r}}. \quad (9)$$

For real coupling devices nothing more than some inconsistency between the real and the second Eq.(4) coefficients, indeterminacy of the reference plane position, the frequency shift from the device reactance take place, since the real device properties are constant in vastly wider band than $[\omega_L, \omega_h]$, Fig. 1a. The last expressions do not contain these factors and can be used for precise tuning.

3 Control signal properties

Consideration of Eq.(6) for the functions[4] $\mathcal{E}_{ac}(t) = \mathcal{E}_a(t) - \mathcal{E}_a(t - T_c)$, $T_c > 0$; $F_c(t) = F(t) - F(t - T_c)$ and also $\int\limits_{-\infty}^{+\infty} F_c(t)dt = 0$ yields

$$\frac{\omega_r^2 \int\limits_0^{\infty}\left(\sum\limits_a \mathcal{E}_{ac}(t)\right)^2 dt}{W_1^2} = \frac{\int\limits_{-\infty}^{+\infty} |F(\omega)|^2\cdot\left|T_c\frac{\sin\frac{\omega T_c}{2}}{\frac{\omega T_c}{2}}\right|^2 d\omega}{2\pi}.$$

The second factor brings out $|F(\omega)|^2$ filtration in $\approx 1/T_c$ bandwidth. The a particle is inside the resonator at $t \in [\tau_{ai}, \tau_{ao}]$; for $\max\{\tau_{ao} - \tau_{ai}\} \ll T_c < \tau_{in}$ with τ_{in} increase the left-hand side integral approximates to $\tau_{in}\cdot\left(\sum\limits_a \mathcal{E}_a(T_c)\right)^2$, for $T_c \gg \tau_{in}$ with T_c increase it approximates to $T_c\cdot\left(\sum\limits_a \mathcal{E}_a(\tau_{ao})\right)^2$, but τ_{in}, T_c

[2]The result is exact for different particle charges also.

[3]This form continuous extension onto $t \in (-\infty; +\infty)$ is correct since the second term in Eq.(7) presents only the ratio of time functions.

[4]The $\mathcal{E}_a(t)$ extension delivers $\int_{-\infty}^{+\infty}|\sum\limits_a \mathcal{E}_a|dt > \infty$, so that Fourier image of the Eq.(6) integral may be nonexistent.

must remain finite. Nevertheless, the definition of (6) spectrums

$$F(\omega)=\frac{\sum\limits_a\left(\mathcal{E}_a(\tau_{ao})e^{-i\omega\tau_{ao}}+i\omega\int\limits_{\tau_{ai}}^{\tau_{ao}}\mathcal{E}_a(t)e^{-i\omega t}dt\right)}{-W_1\omega_r^{-1}} \quad (10)$$

at $\omega = 0$ gives $F(0) = \frac{-\omega_r}{W_1}\sum\limits_a\mathcal{E}_a(\tau_{ao})$. Thus, $\int|F(\omega)|^2 d\omega$ over the low frequency band presents the beam's main characteristics and may be taken as preassigned.

Consider the symmetrical band signal $S_0(\omega) = S(\omega) + S_{ad}(\omega)$, Fig. 1; in the general case $S_0(\omega) = C_0(\omega-\omega_0)e^{i\alpha(\omega-\omega_0)}+C_0(-\omega-\omega_0)e^{-i\alpha(-\omega-\omega_0)}$, where $C_0(y)$, $\alpha(y)$ are defined in $y \in [-y_0,+y_0]$, $y_0 = \omega_0-\omega_L = \omega_{had}-\omega_0$. Now the transformed spectrums (Fig. 1b) coincide in the low frequency band, wherein the squared modulus integration yields $\mathtt{E}\pi = \mathtt{E}_0\pi - \int\limits_{y_b}^{y_0}C_0^2(\omega)d\omega$, where $\mathtt{E}$ is the S signal energy, $y_b = \omega_h - \omega_0$, and the S_0 energy

$$\mathtt{E}_0=\frac{\Lambda}{\pi}-\frac{\int\limits_{-y_0}^{y_0}C_0(\omega)C_0(-\omega)\cos(\gamma)d\omega}{\pi}, \quad (11)$$

where $\Lambda = \int\limits_{-y_0}^{y_0}|F(\omega)|^2 d\omega/A^2$ value is prescribed. Cauchy-Schwarz inequality application determines $\mathtt{E}_0$, $\mathtt{E}$ global minimum at

$$\gamma=2\varphi-\alpha(-\omega)-\alpha(\omega)=0, \quad C_0(\omega)=C_0(-\omega).$$

This conditions are satisfied if $V(\omega)$ in $[\omega_L,\omega_{had}]$ band is the spectrum of an amplitude modulated signal. Usually deviation (8) is much smaller than the required band, $y_b \approx y_0$ and the second term in $\mathtt{E}\pi$ expression is negligible; further analysis assumes symmetrical band $S(\omega)$. In the general case

$$S(t) = A_0(t)\cos(\omega_0 t+\varphi)+B_0(t)\sin(\omega_0 t+\varphi), \quad (12)$$

the $F(\omega)$ compensation (Fig. 1b) in the band $F_0(\omega) = F(\omega)|_{\omega\in[-y_0,+y_0]}$ and in the $F_{+2}(\omega) = F(\omega)|_{\omega\in[2\omega_0-y_0,2\omega_0+y_0]}$ with $F_{-2}(\omega)$ bands[5] proves the realizability of $A_0(t), B_0(t)$ and their spectrums

$$A_0=\frac{-2F_0(\omega)}{A},\ B_0=\frac{2i[F_0(\omega)-2F_{+2}(\omega+2\omega_0)e^{-i2\varphi}]}{A}.$$

Thus, the signal $S(t) = A_{mod}(t)\cos(\omega_0 t + \varphi + \Phi(t))$ contains amplitude and phase modulations

$$A_{mod}(t) = \tfrac{2}{A}\left(\left(\mathbf{F}^{-1}\{F_0(\omega)\}\right)^2 + \left(\mathbf{F}^{-1}\{i(F_0(\omega)-2F_{+2}(\omega+2\omega_0)e^{-i2\varphi})\}\right)^2\right)^{\frac{1}{2}},$$
$$\Phi(t) = \arctan\left(\frac{\mathbf{F}^{-1}\{i(F_0(\omega)-2F_{+2}(\omega+2\omega_0)e^{-i2\varphi})\}}{\mathbf{F}^{-1}\{F_0(\omega)\}}\right); \quad (13)$$

[5] $F(\omega)$ compensation only in the low frequency band is inconsistent because of the "mirror channel" effects.

where $F(\omega)$ is defined by Eq.(10). The signal energy will be minimal at

$$\begin{cases}\Re\{Z\}=1\\ \Im\{Z\}=\text{const}(\omega)\end{cases} \quad Z=\frac{2F_{+2}(\omega+2\omega_0)e^{-i2\varphi}}{F_0(\omega)}, \quad (14)$$

in this case the phase modulation is absent.

The ω_0 harmonic for (7) form is selected at zero correlation of the residual $V(t) - V_0\cos(\omega_0 t + v_0)$ signal with $\cos(\omega_0 t + \varphi)$ $\forall\varphi$ in the finite $V(t)$ interval, at any finite precision $\omega_0\tau_{in} \gg 1$ it yields $v_0 = \arg(V(\omega_0))$, $V_0 = \frac{2}{\tau_{in}}|V(\omega_0)|$; with the use of Eq.(12)

$$V_0=\frac{4|F_{+2}(2\omega_0)|}{A\tau_{in}}, \quad V_0\cos\psi=\frac{2F_0(0)}{A\tau_{in}}. \quad (15)$$
$$v_0=\arg(F_{+2}(2\omega_0))-\varphi,$$

4 Resume

1. Basic idea for the control and the condition Eq.(5) for state equation are obtained. 2. Accelerating resonator external characteristics for RF feed system design are defined, Eq.(9) determine beam's equivalent for the system precise tuning. 3. The reflected wave properties: full reflection at $\omega \neq \omega_0$ with zero reflection on $\omega = \omega_0$ form the possible support for the control. 4. Spectrum $F(\omega)$ (Eq.(10)) completely determines the control signal Eq.(13) and the tuning characteristics Eq.(15,9). 5. Optimal control signal delivers minimum of RF source energy, the signal is simple for realization since it is only the amplitude modulated. The optimum conditions Eq.(14) are also expressed in terms of $F(\omega)$.

The (14) conditions allow to consider RF system optimization.

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