3D SIMULATION STUDY OF ION TRAP IN STORAGE RING

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Abstract

The dynamics of ions trapped in the potential well of electron beams in a storage ring is studied by 3D PIC simulation. The magnetic field profile including fringe field is given by a transverse multi-pole expansion. In this study some numerical results such as the longitudinal reflection of an ion by the fringe field of a bend, the vertical trap of an ion at a bend, and the gradient B drift in undulator fields are shown. The effects of beam parameters such as filling pattern are also included.

INTRODUCTION

In an electron storage ring residual gas molecules are ionized by collisions with electrons circulating in the ring. Such ions are trapped in a potential well made by the circulating electron beam. Through the collision between trapped ions and circulating electrons, beam instability (ion trap instability) could occur and may cause tune shift, blow-up or pulsation of beam size, reduction of beam lifetime, etc [1,2].

The instability has been often analyzed by the simple linear model where it is assumed that there is no magnetic field. However, the model can't explain experimental results well. To understand the ion's effects on the stored electron beam, it is important to calculate ion's dynamics in the realistic and complex filed of the storage ring by numerical simulation.

In this study, simulation is performed with Apple MacPro (OS X, 2.66GHz Quad. Xeon, 4GB RAM). Three-dimensional Particle-In-Cell (PIC) simulation code is written with Intel Fortran, and numerical results are analyzed using visualization software such as Techplot360, Gnuplot and pgplot.

NUMERICAL DATA OF 3D MAGNETIC FIELD

To track the ion's trajectory in the magnetic devices of storage rings such as bend, quadrupole, sextupole and undulator, the calculation of realistic 3D magnetic field is necessary. Especially fringe fields at edges of magnets should be considered for calculation of ion's dynamics..

The 3D magnetic field profiles of dipole, quadrupole and sextupole magnets are provided by a transverse multi-pole expansion [3],

$$B_x(x,y,z)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}C_{m,n+1}(z)\frac{x^n y^m}{n!m!},$$

$$B_y(x,y,z)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}C_{m+1,n}(z)\frac{x^n y^m}{n!m!},$$

$$B_z(x,y,z)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}C^{[1]}_{m,n}(z)\frac{x^n y^m}{n!m!},$$

$$C_{0,0}=0,\qquad C_{2k,n}=\sum_{\ell=0}^{k}(-1)^k\binom{k}{\ell}a^{[2\ell]}_{n+2k-2\ell-1},$$

$$C_{2k+1,n}=\sum_{\ell=0}^{k}(-1)^k\binom{k}{\ell}b^{[2\ell]}_{n+2k-2\ell}$$

$$a_n(z)=\left(\frac{\partial^n B_x}{\partial x^n}\right)\bigg|_{x=y=0}(z),\quad b_n(z)=\left(\frac{\partial^n B_y}{\partial x^n}\right)\bigg|_{x=y=0}(z)$$

In this model, the longitudinal components in varying magnetic region such as fringe field present. The vertical field component of a dipole in a fringe region, for example, is shown in Fig. 1.

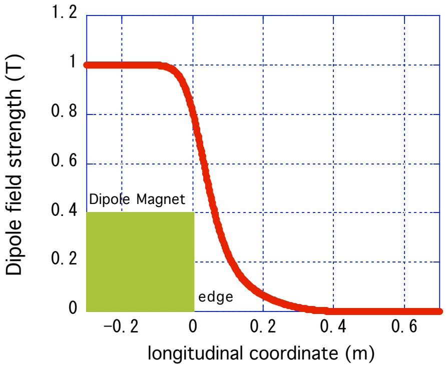

Fig. 1. Calculation of vertical field component of dipole magnet including fringe field.

As for undulator, the magnetic field data is calculated based on the measured data at NewSUBARU Long Undulator (LU).

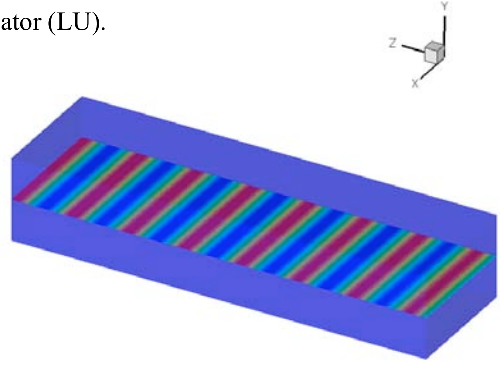

Fig. 2. Calculation region of the undulator field. A slice of vertical field strength at y=0 plane is shown.

PIC SIMULATION CODE

In the simulation code, the three dimensional space is divided into cubic mesh. On each mesh node, magnetic

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fields are evaluated by the multi-pole expansion. Flow of numerical calculation is shown in Fig.3. The coordinate $\boldsymbol{x}_j$ and velocity $\boldsymbol{v}_j$ of the j-th macro-particle can be calculated from the following equations of motion,

$$m\frac{d\boldsymbol{v}_j}{dt} = q\left[\boldsymbol{E}(\boldsymbol{x}_j) + \boldsymbol{v}_j \times \boldsymbol{B}(\boldsymbol{x}_j)\right], \quad \frac{d\boldsymbol{x}_j}{dt} = \boldsymbol{v}_j .$$

In this study, for simplicity, it is assumed that magnetic field is static, i.e., due to only magnetic devices, and electric field due to electrons and ions is given by the following Maxwell equations,

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}, \qquad \rho(\boldsymbol{x}) = \sum_j q n_q(\boldsymbol{x}) .$$

The charge density ρ is evaluated at mesh nodes from density of macroparticle n_q and the distribution of an electron beam.

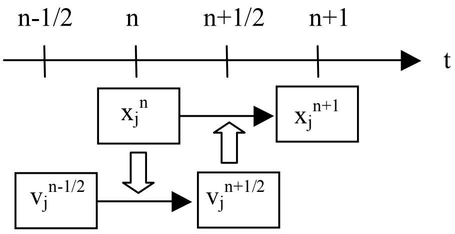

Fig. 3. Flow of calculations of velocity vj and coordinate xj of the j-th macro-particle. xj is calculated at n time step, vj at n+1/2.

EXAMPLE FOR TRACKING RESULTS AND VISUALIZATION

A few results of numerical simulations, for examples, are shown in the following, where typical parameters are those of NewSUBARU storage ring and CO (M=28) is considered as ions

Ion trap in a drift space

To test the validity of the simulation code, the stability of an ion at a magnetic-field-free region is analyzed. The horizontal and vertical coordinates of an ion at various filling patterns are shown in Fig.4. As is well known in the linear theory, the ion is stable in the full-fill and unstable in the single bunch mode. The fact that although the number of bunches is the same the ion is stable in the equidistant filling and unstable in the series filling is in good agreement with the linear theory.

The vertical distribution of the ions trapped in the potential well is shown in Fig.5. Although the initial distribution of two thousands of macro-particles is the same Gaussian as electrons, the trapped ions' distribution well corresponds to the shape of the potential well.

Reflection of an ion in longitudinal direction at dipole fringe field

Assuming an ion generated at field free region and going in longitudinal direction toward a dipole magnet, the more the ion approaches the dipole, the larger the field strength it feels. The motion of the ion is slow down and then the ion is reflected by the magnetic barrier as shown in Fig.6.

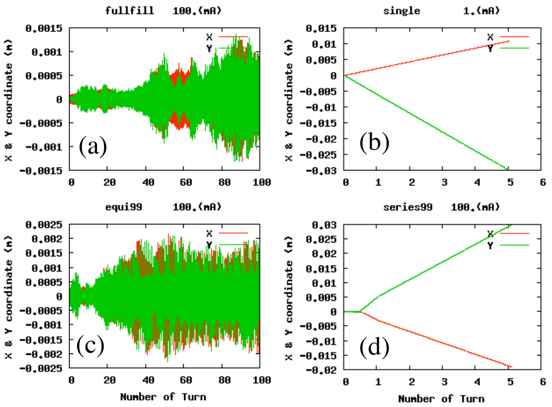

Fig. 4. Horizontal and vertical coordinates of an ion versus the number of revolution turn of electron beams: (a) fullfill at 100mA, (b) single bunch at 1mA, (c) Equidistant 99 bunches at 100mA, (d) Series 99 bunches at 100mA. The harmonic number is 198.

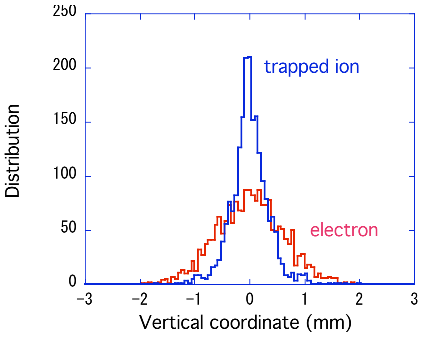

Fig. 5. Vertical distributions of electron beam and trapped ions.

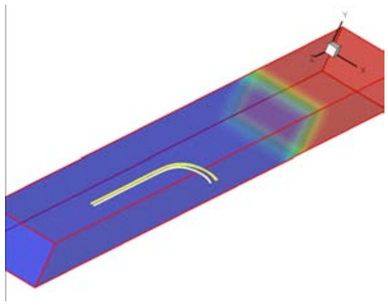

Fig. 6. Calculated trajectories of two ions in dipole magnet. The ions are reflected by the fringe field of the dipole.

Reflection of an ion in vertical direction at dipole field

Assuming off-beam, an ion generated in a dipole field cannot move across the magnetic line of force, but advances in vertical direction along the magnetic line of force, performing the cyclotron motion round the line. However, the more the ion approaches the magnet, the larger the field strength is. By the magnetic barrier effect, the ion is reflected and goes back and forth in vertical direction as shown in Fig.7.

In case of on-beam of both full-fill and single bunch, the time evolution of horizontal and vertical coordinates of an ion are shown in Fig.8. A single bunch rejects the ion as well as in case of Fig.4, although it needs longer time (about 10 turns) compared to the field-free case. In full-fill mode, oscillation amplitudes are larger than the field-free case.

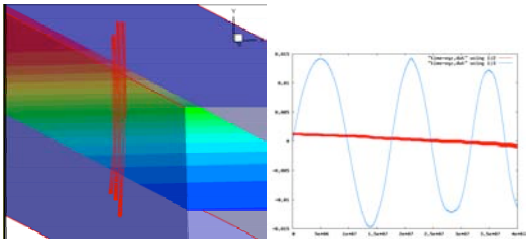

Fig. 7. Calculated trajectory of an ion at the edge of a dipole magnet (off-beam). The ion goes back and forth vertically by magnetic barrier.

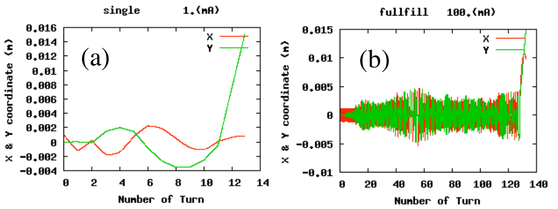

Fig. 8. Horizontal and vertical coordinates of an ion versus the number of revolution turn of electron beams: (a) single bunch at 1mA, (b) full-fill at 100mA.

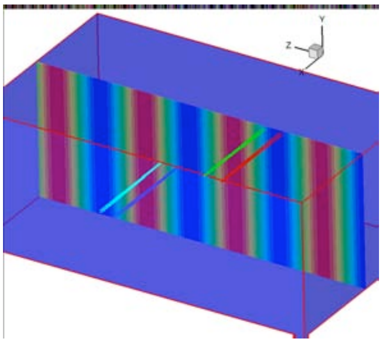

Fig. 9. Calculated trajectories of four ions in undulator field. The ions horizontally escape from the beam axis by $\boldsymbol{B} \times \nabla B$ drift in the case of off-beam. A slice of x=0 plane shows the strength of vertical undulator field.

Drift and trap of an ion in undulator field

In the undulator field, periodic variation of the vertical magnetic field gives rise to the gradient of field ∇B. In off-beam, the ions generated in the undulator field feel forces due to $\boldsymbol{B} \times \nabla B$ drift and moves horizontally in a different direction every half period of the undulator as shown in Fig.9.

Fig. 10 shows calculated trajectories of four ions in case of on-beam with full-fill of 100mA. It can be seen that ions are trapped in both horizontally and vertically by electron beams and also trapped longitudinally between half periods of the undulator.

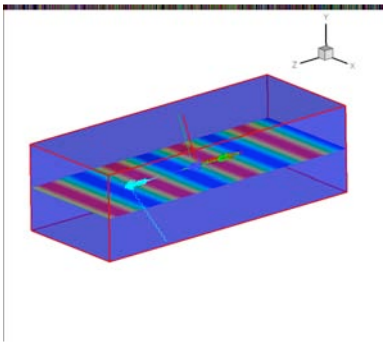

Fig. 10. Calculated trajectories of four ions in undulator field in case of full-fill of 100mA.

SUMMARY

The dynamics of ions trapped in the potential well by electron beam in storage ring is studied by 3D PIC simulation. Magnetic filed profile is provided by multi-pole expansion and exact orbit in complex fileds can be calculated. Ions' dynamics at a bend including fringe field and in undulator field are visually analyzed in both off-beam and on-beam.

A well-programmed code on a multi-core processor makes parallel computing possible, which can make the elapsed time of the simulation task reduced. And visualization software was very useful for data analysis such as 3D trajectory of particles.

In future, this simulation code will be expanded to self-consistently include the effects of trapped ions on electron beams.

ACKNOWLEDGEMENTS

The author thanks to Prof. S. Miyamoto of LASTI for his offer of the measured magnetic field data of LU. This study was supported by Grant-in-Aid form Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology.

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