COMPACT FOUR MIRROR LASER WIRE SYSTEM FOR QUICK MEASUREMENT OF ELECTRON BEAM PROFILE

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Abstract

A compact prototype four mirror optical cavity is being constructed at KEK-ATF to measure a low-emittance electron beam in the damping ring. Length of four mirror rectangular cavity is 103 mm. Minimum beam waist depends on aspect ratio of resonator. In order to achieve the desire value of aspect ratio, distance between adjacent plane mirror and concave mirror are kept very small by using special mirror alignment scheme. Total cavity length of four mirror resonator is matched to pulse repetition of mode locked infra-red laser oscillator. Pulse repetition rate of mode locked laser oscillator is 714 MHz. Electron beam inside ATF Damping Ring has repetition rate of 357 MHz. Minimum beam waist is obtained in sagittal plane using IR pulsed laser. Electron beam scanning time can be reduced using such type of compact laser wire system. With very small laser beam size effective electron –photon collision can be observed. We report the development and performance studies of such type of compact four mirror laser wire system.

1. Introduction

An Accelerator Test Facility (ATF) was built at KEK in hope of developing techniques for the low emittance beam [1]. It consists of an electron linac, a damping ring in which beam emittance is reduced and an extraction line. In a damping ring at ATF, vertical beam size is less than 10 μm. For emittance measurement we are developing a new type of beam profile monitor which works on the principle of Compton scattering between electron and laser light. In order to achieve effective collision of photon and electron, very thin size laser is required. By scanning the position of laser beam and counting the number of scattered photons, a projected beam size is obtained. Such type of optical resonator system is called laser wire. Laser wire is one of such a technique to measure a small electron beam size. If we make a small waist laser beam and install it perpendicular to an electron beam then, electron interacts with the laser light and emits energetic photons in the forward direction by Compton scattering process. After the scattering, all electrons are bent by magnet while emitted photons are detected by a photon detector placed downstream in the forward direction. In particular, if both electron and laser beam are assumed to have Gaussian profiles with width σ_e and σ_{lw} , the observed profile is also gaussian with width σ_{obs} expressed by

$$\sigma_{\mathrm{obs}}^2 = \sigma_{\mathrm{lw}}^2 + \sigma_{\mathrm{e}}^2 \qquad (1)$$

The Boundary conditions for four mirror resonator is set by two concave mirrors and two plane mirrors, which are facing each other [2]. The minimum beam waist is obtained in between two concave mirrors. The two concave mirrors of same curvature are used in compact resonator. Electron beam interacts with laser pulse at minimum beam waist position, which is called interaction point (IP).

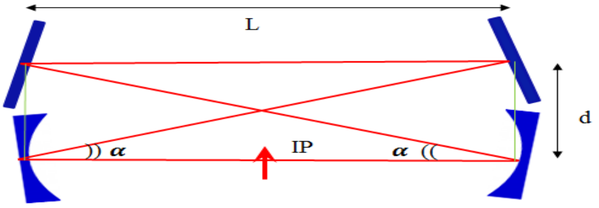

Figure 1: Four mirror resonator model

Minimum beam waist inside four mirror resonator depends on many parameters. Aspect ratio of resonator is defined as ratio of side by side plane and concave mirror distance (d) to distance between two concave mirrors (L). The angle α as shown in Figure 1inside optical cavity is given by

$$\alpha = \tan^{-1} d/L. \qquad (2)$$

If we keep the aspect ratio constant and keep reducing the length of cavity (L) and side by side mirror distance (d), we can achieve very small beam waist. The round-trip time of the cavity has to match with the pulse repetition of the laser source. Hence, precise control of the absolute cavity length is important [3]. In this case, the curvature of the cavity mirrors is the only parameter to control waist size. In order to realize a small spot size, mirrors of specially designed curvature are needed.

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Beam waist is calculated by solving the one round trip ray transfer matrix for beam inside resonator. Distance between the mirrors and curvature of the mirrors are the parameters to design the waist size (w_0) of laser.

The minimum beam waist [4] is given by -

$$\omega = \left(\frac{\lambda}{\pi}\right)^{1/2} \frac{(|B|)^{1/2}}{[1-[(D+A)/2]^2]^{1/4}} \qquad (3)$$

In order to realize the small waist size, the cavity has to to be designed close to the marginally stable configuration [5, 6] .i.e.

$$4-(D+A)^2 > 0,$$

or

$$-2 < (D+A) < 2 \qquad (4)$$

Note that the rms size (σ) of the photon distribution is related to ω_0 as $2\sigma = \omega_0$.

2. Design of compact resonator

2.1 Selection of cavity length and mirror curvature

Assuming a laser oscillation cavity length of L_{laser} and optical cavity length as L_{cav} . Pulsed repetition is the same as the revolution frequency of the laser. All of the frequencies in the mode locked laser can be resonant in the optical cavity when the following condition is satisfied

$$\mathbf{L_{cav}} = \mathbf{n}\frac{\lambda}{2} \qquad (5)$$

$$\boldsymbol{mL_{cav}} = \boldsymbol{L_{laser}} \text{ (m: integer)} \qquad (6)$$

To acieve minimum beam size we have to choose length of resonator “L” close to curvature of optical cavity mirrors.We need to match the resonating frequency of four mirror optical cavity with frequency of oscillation cavity. Therefore total path length of four mirror optical cavity is matched with length of oscillation cavity. The pulse repetition rate of our laser oscillation system is $f_{laser} = 714.037$ MHz. A mode lock pulsed laser consists of an array of superposed waves of many frequency spectrum which provides short pulse lasers. Therefore, when we want to stack the pulsed laser, the resonance frequency of optical cavity has to be matched up with pulsed laser frequency.

The Figure 2 shows beam waist variation with radius of curvature of concave mirror. Horizontal axis indicates radius of curvature of concave mirror in mm and vertical axis shows beam size in mm. To get the minimum beam size in one plane i.e. sagittal plane in this case, we choose values of curvature of cavity mirrors close to the length of resonator. Distance between two concave mirrors are fixed at 103 mm length. In order to obtain minium beam waist in this configuration we keep optical cavity at marginally stable condition and choose value of mirror curvature very close to distance between two concave mirrors. We choose the mirror curvature value as 102 mm.

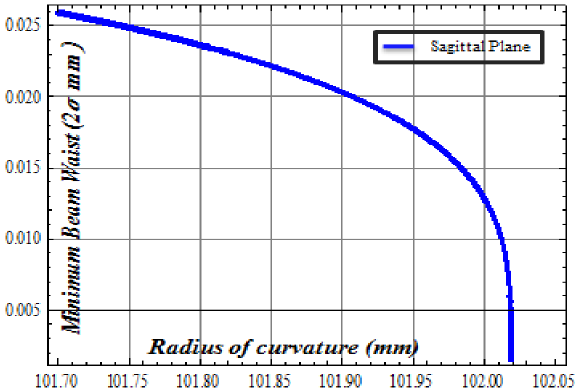

Figure 2: Variation of beam waist with curvature of concave mirror

However, if curvature value is initially fixed , then we can fix the resonator length very close to curvature of mirror. Thus we can obtain minium beam waist configuration.For pulsed resonator we have to keep constant round trip length inside optical cavity. The repetition rate of 714 MHz oscillator is equivalent to 420 mm round trip length (L_{Cav}) inside four mirror cavity. At minimum beam waist position, we obtained elleptical shape beam with smaller sagittal beam waist (σ_S)and larger tangential beam waist (σ_T). Following Table 1 shows values of minimum beam waist in sagittal and tangential plane for different values of curvature of mirror (ρ) and length of cavity (L). Distance “d” between side by side mirrors is kept 29 mm for this setup.

Table 1: Selection of curvature and length of cavity

L(mm)	**ρ (mm)**	**L_{Cav} (mm)**	**σ_S (μm)**	**σ_T (μm)**
103.2	102.2	420.009	7.2	20.01
103	102	420.009	7.3	19.97
102.8	101.8	420.009	7.4	19.94
102.6	101.6	420.009	7.5	19.91

From above table it is clear that, with proper selection of mirror curvature and length of resonator we can approch towards minimum beam waist in sagittal plane.

2.2 Mirror Alignment Scheme

It is very difficult to keep the center to center adjacent mirror distance of 29 mm for all 1 inch diameter mirrors. It requires a complicated mirror holder design to keep small distance “d”. The distance between adjacent concave mirror and plane mirror is reduced by using aluminum spacers. These spacers provide the function to keep all the mirrors at the slope of 45 degree angle. Therefore it is possible to reduce the distance between side by side plane and concave mirror to desired value. Such type of structure is shown in Figure 3.

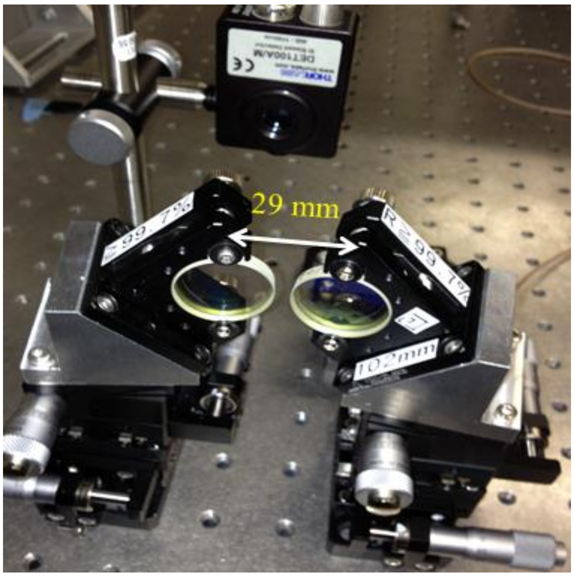

Figure 3: Tilted mirror holders on aluminum spacers

2.3 Beam Evolution inside resonator

The Figure 4 shows the evolution of beam size in both sagittal and tangential plane along longitudinal distance inside four mirror resonator. The beam size is squeezed in between concave mirrors. Large value of beam waist is obtained at the interface of concave mirrors. Smaller beam size in between two conacve mirrors depends on the divergence of beam inside resonator. If beam waist at the interface of concave mirror is large then we get smaller value of beam waist in between two concave mirrors. The beam size is almost constant in both planes while prapogating through free space region formed among concave-plane-plane-concave mirrors. Beam size at the surface of concave mirrors and plane mirrors are greater than 1.2 mm in sagittal plane , and greater than 0.4 mm in tangential plane. Selection of mirror diameter is quiet imporatnt in this situation in order to avoid any diffraction of laser beam around edges of mirror. All mirrors used in compact resonator setup are of 1 inch diameter.

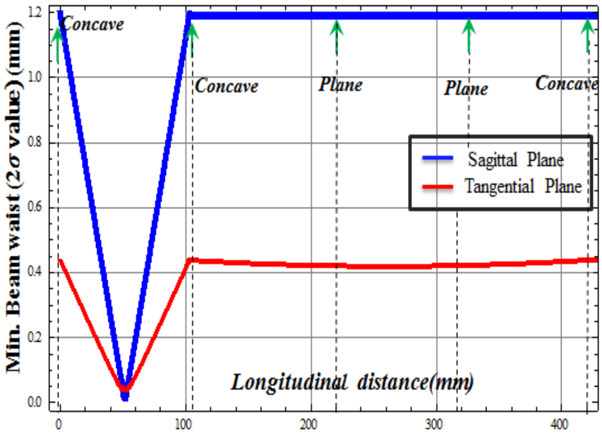

Figure 4: Beam evolution inside four mirror optical cavity

Another important thing is shape of beam at the injection plane mirror. It is clear from Figure 4 that sagittal size is greater than tangential size at the interface of mirror. It is property of optical cavity to produce a beam which has larger beam size in sagittal plane as compare to beam sixe in tangential plane after a round trip of laser beam inside cavity. At mirror interface, the ratio of beam size in sagittal plane (1.2 mm) to beam size in tangential plane (0.4 mm) is approximately equal to 3. There must be good coupling of oscillator photon and round trip cavity photon at injection mirror for good storage power inside resonator. Therfore, injection beam must have sagittal to tangetial beam size ratio of 3 for good coupling. Laser beam is injected through a plane mirror inside optical cavity. Cylindrical lense system is used to convert circular beam into elleptical beam and spherical lense system is used to reduce the divergence of laser beam.

3. Parameters of compact resonator

Enhancement Factor and Finesse

When a laser beam is injected into four mirror optical cavity from one end, a part of beam comes inside the cavity and reflects back and forth among four mirrors. This phenomenon is illustrated in Figure 5. The amplitude of each optical path can be calculated by considering the reflection and transmission experience at the interface of mirrors. Important role of the cavity is to enhance the effective laser power. Laser beam from a laser oscillator is injected to the cavity through one of the plane mirrors. The laser wave inside the cavity reflects back and forth and builds up the effective power. The power enhancement realizes only when the cavity satisfies the resonance condition of a standing wave. We define mirror reflectivity for the laser amplitudes as r_1, r_2, r_3 and r_4. Mirror transmittance for laser amplitudes are defined as t_1, t_2, t_3 and t_4. Mirror reflectivity and transmittance parameters defined for the laser power are R_1, R_2, R_3, R_4 and T_1, T_2, T_3, T_4. The relation between these two definitions can be expressed as $r = \sqrt{R}$, $t = \sqrt{T}$. We assume injection beam at plane mirror has unity amplitude.

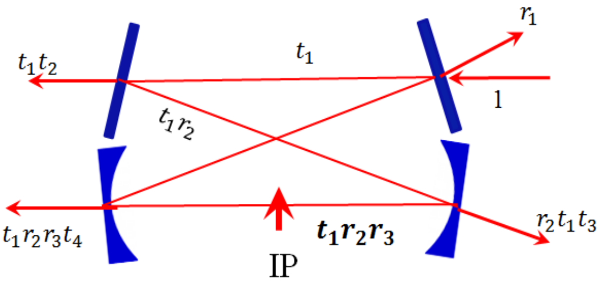

Figure 5: Laser reflectivity and transmittance amplitudes inside optical cavity

We consider the power enhancement at IP, because finally this is the point where electron beam and laser beam collide with each other. The enhancement factor (S_{Cav}) at IP is given by

$$S_{Cav} = \frac{T_1 R_2 R_3}{(1-R_{eff})^2} \qquad (7)$$

Where R_{eff} is effective reflectivity of resonator defined by

$$R_{eff} = \sqrt[4]{R_1 R_2 R_3 R_4} \qquad (8)$$

Sharpness of the resonance width is represented by the cavity finesse (*F*), it is defined from the reflectance of the four mirrors of optical cavity as [4, 7]

$$F = \frac{\pi R_{eff}}{1-R_{eff}^2} \qquad (9)$$

Design reflectivity (R_1 and R_2) of plane mirrors are 99.67% and 99.985%. Reflectivities of both concave mirrors (R3 and R4) are 99.985 %. Total Finesse (F) of compact resonator is given by [2]

$$F = \pi \frac{\sqrt[4]{R_1 R_2 R_3 R_4}}{1-\sqrt{R_1 R_2 R_3 R_4}} \qquad (10)$$

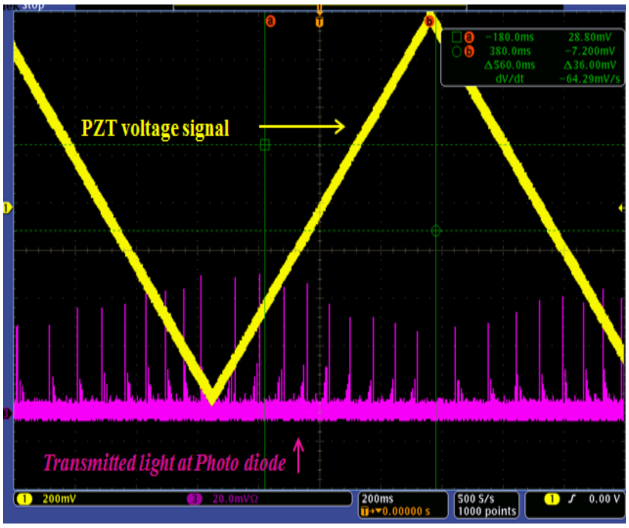
PZT voltage signal →
Transmitted light at Photo diode ↑

Figure 6: Transmitted laser signal

Finesse is measured experimentally by finding the ratio of Free Spectral range (FSR) to width of resonance at half maximum ($\delta\theta$) of Airy function. FSR is distance between peaks of two consecutive 0th order modes. In Figure 6, the yellow waveform shows the voltage of piezo actuator, which means cavity length expansion. The red wave form shows the signal from photo diode detecting the cavity transmitted laser power.

$$\text{Experimental Finesse} = FSR/\delta\theta \qquad (11)$$

Experimental Finesse is calculated as 1407 ± 28. Enhancement Factor (S_{Cav}) is calculated as 710.2

4. Beam Waist Measurement

4.1 Divergence Method

The output laser profile is an extension of the cavity resonating mode, so the waist size of laser beam inside optical cavity can be determined by measuring the output profile by scanning the pin hole photo diode, both in horizontal plane and vertical plane as shown in Figure 7.

The output laser size at the distance z from the focal point is represented by $\omega(z) = \omega_0\sqrt{1+(z/z_0)^2}$, Where z_0 is the Rayleigh length. In the case of z≫ z_0, the divergence angle θ_0 can be approximated as [8]:

$$\theta_0 = \omega(z)/z = \lambda/\pi\omega_0 \qquad (12)$$

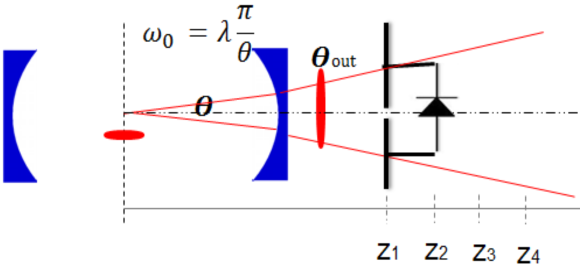

Figure 7: Minimum beam waist measurement using divergence method

The minimum beam waist measured using divergence method in sagittal plane (σ_S) is 10.37± 1.73 μm and in tangential plane (σ_T) is 23.65 ± 2.82 μm.

4.2 Guoy Phase Difference Method

The phase advance in one round trip should be integers of π to be a resonance condition. It depends on not only the cavity length but also Guoy phase. The Guoy phase is defined by the order of the transverse mode (m+n) and the beam waist (ω_0) [9, 10]. The distance between two modes of one-order difference is defined by beam waist. It is illustrated in Figure 8. When cavity length was swept while monitoring the resonation by the cavity transmission intensity, some peaks of the resonances were observed. Each resonance peak corresponds to some order of modes. There are two 1st order modes representing two values of Guoy phase corresponding to sagittal and tangential plane. Guoy phase plays an important role in determining the position and slope of intra-cavity rays and spectral properties of mode. Guoy phase 1 and Guoy phase 2 are defined as distance between 0th and 1st order peaks. FSR is average distance between two consecutive fundamental modes. In order to calculate minimum beam waist of resonator based on mode difference method, distance between plane –plane and concave –concave mirrors are changed while keeping the total length of pulsed resonator constant.

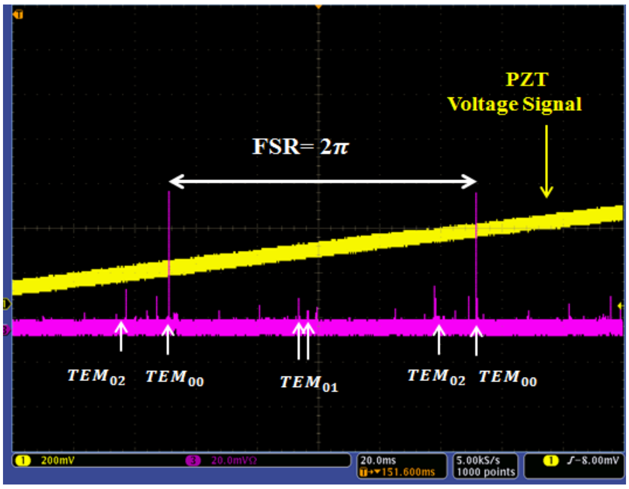

Figure 8: Mode diffrence method

Guoy phase can also be represented by ray transfer matrix of resonator for one round trip. Eigen values of a non-degenerate matrix are complex [11] and are given by

$$m_1 = m_2^* = e^{i\varphi} \qquad (13)$$

$$e^{\pm i\varphi} = \frac{A+D\pm\sqrt{(A+D)^2-4}}{2} \qquad (14)$$

Where phase angle φ is round trip Guoy phase of resonator. Guoy phase is calculated with every change in cavity length and minimum beam waist information is extracted. Figure 9 shows the variation of Guoy phase in sagittal and tangential plane with mirror separation.

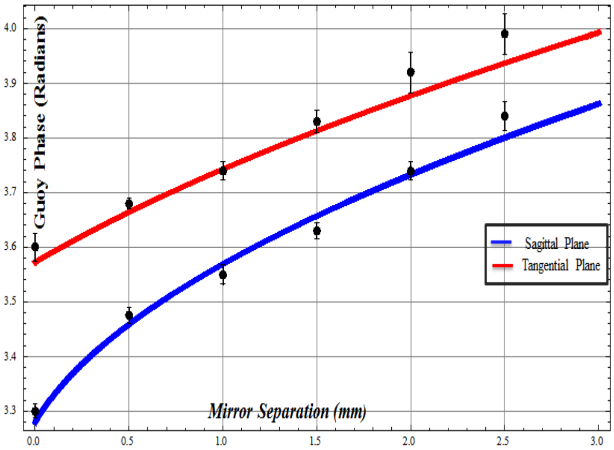

Figure 9: Variation of Guoy phase with mirror sepration

Minimum beam waist measured using Guoy phase difference method in sagittal plane (σ_s) is 13.74± 1.84 μm and in tangential plane (σ_T) is 21.8 ± 3.6 μm.

5. Conclusion

Using IR pulsed compact four mirror resonator, we can obtain very small minimum beam waist in sagittal plane. The results of beam waist measurement using Guoy phase difference method and divergence method are comparable. It is found that minimum beam waist of compact resonator has very high sensitivity towards any change in cavity length. A special mirror alignment scheme is needed to keep side by side distance between concave and plane mirror to 29 mm. Electron beam can be measured in vertical, horizontal and longitudinal direction in very short time as compare to CW laser wire system [12]. Four mirror resonator has less sensitivity for misalignment compare to two mirror cavity. If we carefully select length and mirror curvature parameters, small size beam waist can be achieved.

Acknowledgement

This research has been supported by Quantum beam technology program of Japanese Ministry of Education, Culture, Sports, Science and Technology (MEXT).

REFERENCES

[1] F. Hinode, S. Kawabata, H. Matsumoto, K. Oide, K. Takata, S. Takeda, and J. Urakawa, KEK Internal Report No. 95-4, 1995.
[2] Y. Honda et al., Opt. Commun. 282 (2009) 3108.
[3] H. Shimizu, S. Araki, Y. Funahashi, Y. Honda, T. Okugi, T. Omori, N.Terunuma, J. Urakawa, M. Kuriki, S. Miyoshi, T. Takahashi, Y. Ushio, T.Hirose, K. Sakaue, M. Washio, P. Guoxi, and L. XiaoPing, J. Phys. Soc. Jpn, 78, 074501 2009.
[4] Amnon Yariv, Pochi Yeh, Photonics-Optical Electronics in Modern Communication, 6th edition, New York, 2007
[5] Norman Hodgson, Horest Weber, Optical Resonators, Springer, 1997, pp. 41-42.
[6]A.E. Siegman, LASERS, 1986
[7] Y. Honda et al, Nucl. Instr. and Meth. A 538 (2005) 100-115.
[8] Y. Sakamura, Y. Hemmi, H. Matsuo, H. Sakai, N. Sasao, Y. Higashi, T. Korhonen, T. Taniguchi, and J. Urakawa, hep-ex/9907054
[9] H. Sakai, Doctoral Thesis, Kyoto University, 2002, submitted; http://wwwhe.scphys.kyotou.ac.jp/paper/index.html
[10] H. Sakai, et al., Jpn. J. Appl. Phys. 41 (2002) 6398.
[11] Steven J. M. Habraken , Gerard Nienhuis, Modes of a twisted optical cavity, Physical Review A **75**, 033819 (2007)
[12] J. Urakawa, K. Kubo, N. Terunuma, T. Taniguchi, Y. Yamazaki, K. Hirano, M. Nomura, I. Sakai,M. Takano, N. Sasao, Y. Honda, A. Noda, E. Bulyak, P. Gladkikh, A. Mytsykov, A. Zelinsky, F.Zimmermann, Phys. Rev. A 532 (2004) 388-393.