

Analysis of the 50 Hz Rapid Cycle Power Supply

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ABSTRACT

As for two ring collider of 8 GeV electron and 3.5 GeV positron, B factory under discussion at KEK, the booster ring (injector ring) is planned to be a rapid cycle synchrotron having 50 Hz frequency. Magnet current excited by the repetitive pulse power supply through a choke transformer is analyzed and its operational properties are examined.

INTRODUCTION

For the project of two ring collider for B physics, rapid cycle booster synchrotron will be used as an injector. Booster also comprises two rings which accelerate electron and positron up to 8 and 3.5 GeV, respectively. These booster rings are planned to operate at 50 Hz to fill the collider rings in an attainable short time.

So far the rapid cycling synchrotron consists of one secondary current circuit in which all magnets are connected in series but divided into several groups (meshes) to compensate ac power loss through the choke transformers¹⁻³). Even in this case the common current flows in all magnets.

In a slow cycling synchrotron, bending magnets and quadrupole magnets are powered by separate power supplies composing many current circuits. Currents of focusing and defocusing quadrupoles are adjusted independently to get better performance. Moreover, tuning becomes very flexible if the local quadrupoles can be adjusted individually.

In the case of synchrotron with many current circuits, synchronization of magnetic fields of different circuits is essential. It is realized in the slow cycling synchrotron but not in the fast cycling synchrotron. Therefore it is required to study whether multi-circuits are adaptable to the rapid cycle operation or not.

ANALYTICAL TREATMENT OF 1 MESH CIRCUIT

For the sake of simplicity 1 mesh circuit of Fig.1 is treated analytically. Assuming an ideal transformer, voltage induced in the secondary windings is a multiple of the primary voltage by the winding ratio. Then, the circuit equations are,

$$\text{primary; } \frac{V_s}{n} = \frac{L_{ch}}{n} \frac{dI}{dt} + \frac{L_{ch}}{n^2} \frac{dI_p}{dt} \quad (1)$$

$$\text{secondary; } R_{ch}I + \frac{L_{ch}}{n} \frac{dI_p}{dt} + L_{ch} \frac{dI}{dt} + \frac{1}{C} \int I_1 dt \quad (2)$$

$$R_m I_2 + L_m \frac{dI_2}{dt} - \frac{1}{C} \int I_1 dt = 0 \quad (3)$$

$$I = I_1 + I_2 \quad (4)$$

If there is no coupling of magnetic induction from secondary to primary windings of transformer, the following equation is obtained from (1) and (2).

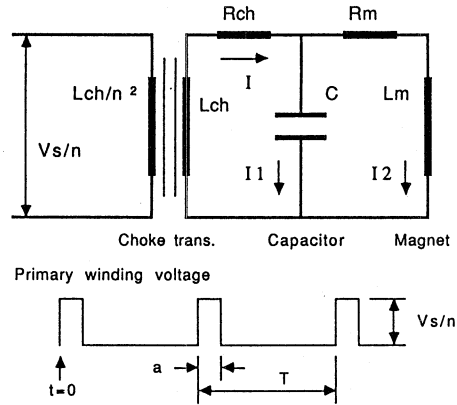


Fig.1 Skeleton of one mesh circuit.

$$R_{ch} + V_s L_{ch} \frac{dI}{dt} + \frac{1}{C} \int I_1 dt = 0 \quad (5)$$

By Laplace transformation (3), (4) and (5) become

$$L\{V_s\} + (R_{ch} + sL_{ch})L\{I\} + \frac{1}{sC}L\{I_1\} = 0 \quad (6)$$

$$(R_m + sL_m)L\{I_2\} - \frac{1}{sC}L\{I_1\} = 0 \quad (7)$$

$$L\{I\} = L\{I_1\} + L\{I_2\} \quad (8)$$

Generality is not lost by assuming for simplicity

$$\frac{R_m}{L_m} = \frac{R_{ch}}{L_{ch}} \quad (9)$$

And resonant angular frequency ω is defined as

$$\omega^2 = \frac{L_m + L_{ch}}{L_m L_{ch} C} \quad (10)$$

Then, $L\{I_1\}$ and $L\{I_2\}$ are

$$L\{I_1\} = -\frac{s}{L_{ch}} \frac{L\{V_s\}}{s^2 + s \frac{R_{ch}}{L_{ch}} + \omega^2} \quad (11)$$

$$L\{I_2\} = \frac{L\{I_1\}}{s(s + \frac{R_m}{L_m}) L_m} \quad (12)$$

As V_s may be assumed as a periodic function of square pulse with a period of $T = \frac{1}{f}$ ($= 0.02$ sec for 50 Hz),

$$L\{V_s\} = \frac{V_s}{1 - e^{-sT}} \frac{1 - e^{-as}}{s} \quad (13)$$

where a is the pulse width of V_s . From (11) and (13),

$$L\{I_1\} = -\frac{V_s}{L_{ch}} \frac{1}{s^2 + s \frac{R_{ch}}{L_{ch}} + \omega^2} \frac{1 - e^{-as}}{1 - e^{-sT}} \quad (14)$$

is derived. I_1 is obtained by the inverse Laplace transformation of (14) applying the second shifting theorem as follows,

$$I_1(t) = f(t) - u(t-a)f(t-a) + u(t-T)f(t-T) - u(t-T-a)f(t-T-a) + \dots \quad (15)$$

where u is a unit step function and

$$f(t) = -\frac{V_s}{\omega_1 L_{ch}} e^{-\frac{R_{ch}}{2L_{ch}} t} \sin(\omega_1 t). \quad (16)$$

ω_1 is defined as

$$\omega_1^2 = \omega^2 - \frac{1}{4} \left(\frac{R_{ch}}{L_{ch}} \right)^2 \quad (17)$$

I_2 is obtained similarly.

$$I_2(t) = g(t) - u(t-a)g(t-a) + u(t-T)g(t-T) - u(t-T-a)g(t-T-a) + \dots \quad (18)$$

where

$$g(t) = -\frac{V_s}{L_m + L_{ch}} \left[\frac{L_m}{R_m} \left(1 - e^{-\frac{R_m}{L_m} t} \right) - \frac{1}{\omega_1} e^{-\frac{R_{ch}}{2L_{ch}} t} \sin(\omega_1 t) \right]. \quad (19)$$

Currents in capacitor and magnet are given by (15) and (18), respectively. And the current of choke transformer is given by their sum. Buildup of currents are demonstrated in the computer simulation as in Fig.2, in which parameters were assumed as $f = 50$ Hz, $L_{ch} = 0.0678$ H, $L_m = 0.044$ H, $R_{ch} = 0.234 \Omega$, $R_m = 0.152 \Omega$ and $C = 0.38$ mF. Ten times larger values were used for both R_{ch} and R_m , because it took too long time to attain a steady state for smaller values. In a real case of smaller resistivities, the buildup time is estimated to be a few minutes.

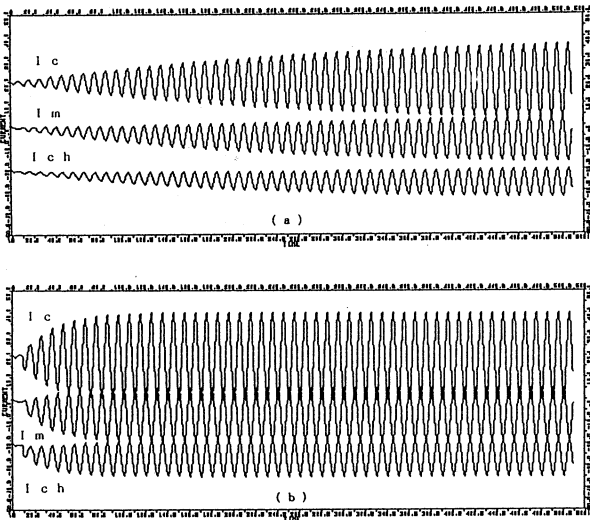


Fig.2 Buildup of 50 Hz currents of capacitor (I_c), magnet (I_m) and choke transformer (I_{ch}). Each current cycle is plotted at (a) every cycle and (b) every 10 cycles.

If the period of excitation pulse V_s differs from that of the resonant frequency of the secondary circuit, the steady state

peak currents decreases as shown in Fig.3. To maintain the same peak current it is required to increase V_s . Shift of a resonant angular frequency ω occurs by the change of inductance and/or capacitance due to the ambient temperature.

Sudden change in periodicity of V_s gives rise the gradual buildup of current oscillation with the same period. But it is so slow that small occasional variation of periodicity may be allowed. The same thing may be said to the amplitude of V_s .

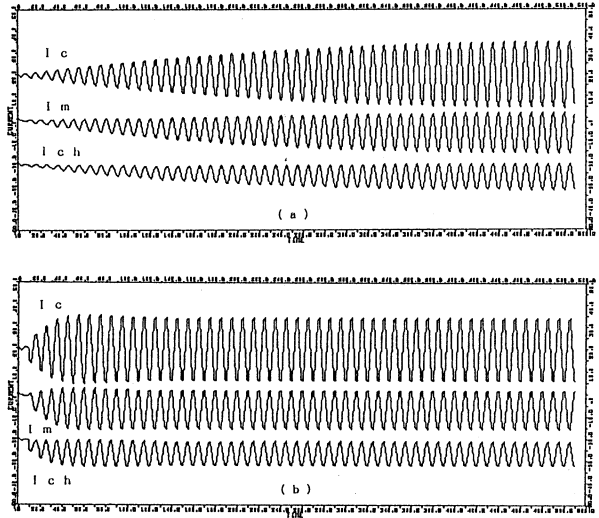


Fig.3 Decrease of steady state current when the repetition frequency of V_s differs by 0.29 Hz. Each current cycle is plotted at (a) every cycle and (b) every 10 cycles.

MULTI-MESH CIRCUIT

Three mesh circuit was treated by Fox¹⁾ for the case that every choke transformer had a mutual inductance. In a general case more than 3 meshes (Fig.4), distributed choke transformers are feasible because of structural complexity in a united type in which all mesh choke transformers are built in and couples. Mutual inductance between choke transformers does not exist in the distributed choke and the circuit equations are

$$L_{m1} \frac{dI_{m1}}{dt} + L_{m2} \frac{dI_{m2}}{dt} + \dots + L_{mn} \frac{dI_{mn}}{dt} + \frac{1}{C_1} \int I_{c1} dt + \frac{1}{C_2} \int I_{c2} dt + \dots + \frac{1}{C_n} \int I_{cn} dt = 0 \quad (20)$$

$$L_{chj} \frac{dI_{chj}}{dt} + V_{sj} = \frac{1}{C_j} \int I_{cj} dt \quad (j=1,2,\dots,n) \quad (21)$$

$$I_{mj} = I_{chj} + I_{cj} \quad (j = 1, 2, \dots, n) \quad (22)$$

and

$$I_{m1} = I_{m2} = \dots = I_{mn} = I_m. \quad (23)$$

For the sake of simplicity, only the case of no power loss is treated. Similar to the case of one mesh, if $L\{I_m\}$ and $L\{V_{sj}\}$ are Laplace transformations of magnet current $I_m(t)$ and induced voltage in secondary j -th mesh windings V_{sj} , respectively,

$$\begin{aligned} & s \left[\sum_j L_{mj} + \sum_j \frac{L_{chj}}{s^2 L_{chj} C_j + 1} \right] L\{I_m\} \\ & = - \sum_j \frac{L\{V_{sj}\}}{s^2 L_{chj} C_j + 1} \end{aligned} \quad (24)$$

It is possible to adjust C_j so that

$$L_{ch1} C_1 = L_{ch2} C_2 = \dots = L_{chn} C_n = L_{ch} C. \quad (25)$$

Then,

$$\begin{aligned} & s[(s^2 L_{ch} C + 1) \sum_j L_{mj} + \sum_j L_{chj}] L\{I_m\} \\ & = - \sum_j L\{V_{sj}\}. \end{aligned} \quad (26)$$

Therefore,

$$L\{I_m\} = - \frac{\sum L\{V_{sj}\}}{L_{ch} C \sum L_{mj}} \frac{1}{s(s^2 + \omega^2)} \quad (27)$$

where

$$L\{V_{sj}\} = \frac{\sum V_{sj} (1 - e^{-as})}{s(1 - e^{-sT})} \quad (28)$$

and

$$\omega^2 = \frac{\sum L_{mj} + L_{chj}}{L_{ch} C \sum L_{mj}} + \frac{1}{L_{ch} C}. \quad (29)$$

Then, Laplace transformation of magnet current I_m is given by

$$L\{I_m\} = - \frac{\sum V_{sj}}{L_{ch} C \sum L_{mj}} \frac{1}{s^2(s^2 + \omega^2)}, \quad (30)$$

where the periodic term $(1 - e^{-as}) / (1 - e^{-sT})$ is omitted because of the same treatment as before. By the inverse transformation,

$$I_m(t) = - \frac{\sum V_{sj}}{L_{ch} C \sum L_{mj}} \frac{1}{\omega^2} \left(t - \frac{\sin \omega t}{\omega} \right) \frac{1 - e^{-as}}{1 - e^{-sT}} \quad (31)$$

and

$$I_{ck}(t) = \left(-D \sin \alpha t + B \sin \omega t + B \frac{\sum L_{mj}}{\sum L_{chj}} \sin \omega t \right) \frac{1 - e^{-as}}{1 - e^{-sT}} \quad (32)$$

($k = 1, 2, \dots, n$),

where

$$B = \frac{\sum V_{sj}}{\omega^3 L_{ch} C \sum L_{mj}} \quad (33)$$

$$D = \frac{1}{\alpha} \left[\frac{\sum V_{sj}}{(\omega^2 - \alpha^2) L_{ch} C \sum L_{mj}} - \alpha^2 C_k V_{sk} \right] \quad (34)$$

and

$$\alpha^2 = \frac{1}{L_{ch} C}. \quad (35)$$

For symmetrical meshes, $D=0$. $I_c(t)$ and $I_m(t)$ are identical to the solutions of 1 mesh circuit if $R_m = R_{ch} = 0$ is assumed.

In an asymmetric case,

$$I_m(t) = -Ft + B \sin \omega t \quad (36)$$

$$I_{cj}(t) = -D \sin \alpha t + B \sin \omega t + B \frac{\sum L_{mj}}{\sum L_{chj}} \sin \omega t \quad (37)$$

($j = 1, 2, \dots, n$),

where

$$F = \frac{\sum V_{sj}}{\omega^2 L_{ch} C \sum L_{mj}} \quad (38)$$

and

$$F, B, D, V_{sj} > 0. \quad (39)$$

The term $-Ft$ in (36) is due to the periodic excitation by V_{sj} in the lossless circuit. This term disappears if ac loss is introduced. So the second term $B \sin \omega t$ is the alternating current in the magnet. Current in the secondary windings of the choke transformer is given by

$$I_{chj}(t) = I_m(t) - I_{cj}(t) \quad (40)$$

and is expressed by

$$B \frac{\sum L_{mj}}{\sum L_{chj}} \sin(\omega t) - D \sin(\alpha t). \quad (41)$$

Differences peculiar to each mesh such as capacitor and coupling between the primary and secondary windings result in D which determines the spurious-mode current closed in each tank circuit. Magnet current does not deteriorate by the spurious-mode current.

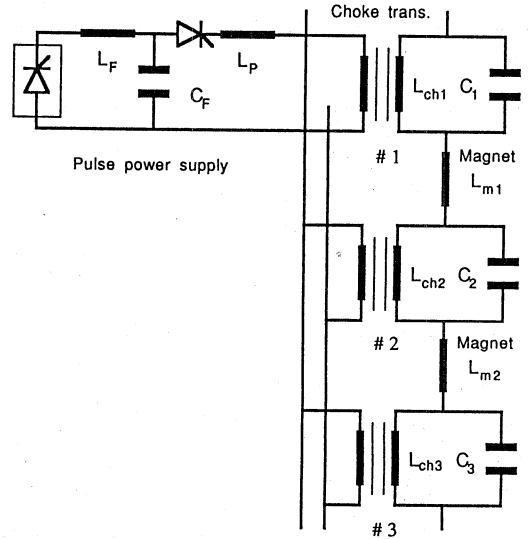


Fig.4 Multi-mesh circuit.

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