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A RESONANT EXTRACTION TECHNIQUE FOR A COMPACT SYNCHROTRON; ASYMMETRIC DRIVING OF TWO SEXTUPOLE MAGNETS

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Abstract

A technique for third-order resonant extraction for a compact synchrotron is presented. Two sextupole magnets are asymmetrically drived for two purposes, one of which is to excite the third-order resonance and the other is to correct for chromaticity. The positions of these two sextupole magnets are decided so that the outgoing branches of the separatrices at the deflector are superimposed for all momenta. This arrangement is effective to realize a compact synchrotron, which needs less space to install magnets, and has a high efficiency of extraction and good quality of the extracted beam.

1. Introduction

Resonant extraction methods have been widely applied not only to physics experiments but also to medical applications [1,2]. The resonant extraction utilizes a nonlinear resonance of the betatron oscillations. A nonlinear magnetic field brings about the resonance and makes the separatrix, which is defined as the boundary between stable and unstable betatron oscillations in the phase space. The separatrix size decreases as the deviation of the tune from the resonant condition becomes smaller. As the tune depends on the particle momentum through a finite chromaticity, the separatrix size also depends on it. The dependency of the separatrix size on the particle momentum causes a change in the orbit gradient of the extracted beam at the deflector position. Any change of beam characteristics can lower the extraction efficiency, which is sometimes disadvantageous for the beam user. Usually, it is necessary to control the chromaticity during the extraction procedure.

When a synchrotron utilizes the third-order resonant extraction, two sets of sextupole magnets have been installed, one of which act as the corrector of the chromaticity, while the other act as the exciter of the third-order resonance. Two sextupole magnets must be installed at a symmetrical position in the synchrotron as the corrector of the chromaticity and drived of same strengh so as to cancel the contribution to the third-order resonance each other. As a result, at least three sextupole magnets, one exciter and two correctors, are needed. Frequently, four sextupole magnets, two exciters and two correctors, are installed to control the chromaticity and resonance independently.

In order to realize a high quality beam or flexible control, synchrotrons such as for accelerator physics or high energy physics must have many components. These synchrotrons become large in order to get enough space for the components and operations may become rather complicated. The size of the synchrotron should be made as small as possible to reduce the cost, including both the machine construction and the civil engineering aspects [3]. It is necessary to develop a method which realizes a high quality beam with fewer components.

This paper proposes a technique for third-order resonant extraction with the chromaticity correction for a compact synchrotron. Two sextupole magnets for the exciters of the third-order resonance serve simultaneously as the correctors of the chromaticity. In order to reduce the extracted beam emittance, the positions of these two sextupole magnets are decided so that the outgoing branches of the separatrices at the deflector are superimposed for all momenta. By combining the above technique with the extraction scheme under a constant separatrix [4], it is expected that a low emittance beam can be easily extracted with a high efficiency.

2. Third-Order Resonant Extraction with the Chromaticity Correction

At the third-order resonance, the stable areas are triangles and the separatrices are straight lines (Fig.1). We consider the third-order resonance driven by a single sextupole of normalized strength Sn given by:

$$S_n = \frac{\beta^{1.5}}{2} S \quad \text{with } S = \int \frac{B''}{B\rho} dl \qquad (1)$$

where $B\rho$ is the magnetic rigidity of the beam and β is the horizontal betatron amplitude function at the sextupole magnet. The integral of the second horizontal derivative of the vertical component of the magnetic field, B", is evaluated over the length of the sextupole magnet.

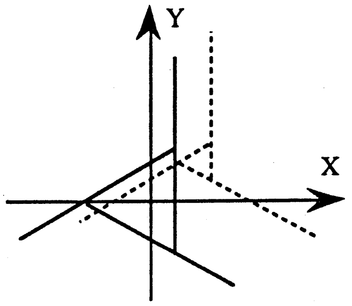

Fig.1 Separatrices of the third-order resonance in the normalized phase space. Dotted lines show the separatrix for an off momentum particle.

More generally, we have to consider the third harmonic of the sextupole magnetic field distribution in the betatron phase around the machine. When there are m sextupoles of strength S_i and normalized strength S_{ni} at phase ϕ_i, the resulting effect of these sextupoles has a strength S and a phase ϕ with:

$$S\exp(3j\phi)=\sum_{i=1}^{m}S_{ni}\exp(3j\phi_i)=\sum_{i=1}^{m}\beta^{1.5}S_i\exp(3j\phi_i)\ . \quad (2)$$

In this equation, phases are taken from the considered point. In general, according to the difference of the phase advance from each sextupole ϕ_i, the resulting phase ϕ changes by each normalized sextupole strength S_{ni}. Some synchrotrons utilize this character to rotate the separatrix and change the orbit gradient of the extracted beam [5]. When the phase difference between each sextupole is $n\pi/3$, the resulting phase does not change for strength of each sextupole and then the separatrix does not rotate.

In the following study, we restrict ourselves to the case of a ring which has superperiods of even number and two sextupole magnets installed on opposite sides of the ring. The phase difference between the two sextupole magnets is nearly $n\pi/3$, because one turn phase advance is $2n\pi/3$, near the resonance condition. As the Twiss parameters at these sextupole positions are equal, the separatrix size ε and the chromaticity ξ (defined as $(\Delta\nu/\nu)/(\Delta p/p)$) become:

$$\varepsilon=\frac{\pi}{\sqrt{3}}\left(\frac{24\delta}{S_1\pm S_2}\right)^2 \quad (3)$$

$$\xi=\xi_{nat}+\frac{1}{4\pi\nu}\beta\eta(S_1+S_2) \quad (4)$$

where S_1 and S_2 are the strengths of these two sextupole magnets, β and η are the betatron and dispersion functions (defined as $\Delta x\Delta p/p$) at these sextupole magnets. ν and δ are the tune and its deviation from the resonant condition. ξ_{nat} is the part of the chromaticity without the sextupole magnets contribution, natural chromaticity. In Eq. (3), the plus (minus) sign is taken when the number n of the phase difference between the two sextupole magnets, $n\pi/3$, is even (odd). When this sign is minus, the separatrix size and the chromaticity can be tuned independently by selecting S_1 and S_2 carefully.

The difference between the strengths of these two sextupole magnets becomes large when a rather large chromaticity change is needed. Then the strength of one of the sextupole magnets becomes strong. In this case, trajectories of the extracted particles curve in the phase space due to the higher order term of the nonlinear resonance. Trajectories curved like this are not suited to the extraction. So this technique is not proper for a ring of large chromaticity.

3. Superimposing the Separatrices at the Deflector

In order to reduce the extracted beam emittance, i.e., the phase space area of the extracted particles at the deflector position, the outgoing branches of the separatrices are superimposed. This condition is derived for the ring of finite chromaticity [6]. In this study, the chromaticity is corrected by the technique shown above.

The normalized horizontal phase plane (X,Y) is defined as:

$$X=\frac{x}{\sqrt{\beta}} \quad \text{and} \quad Y=\frac{\alpha x+\beta x'}{\sqrt{\beta}}$$

where x and x' are the real horizontal position and orbit gradient respectively, and β and α are the Twiss parameters at the considered point.

When the phase difference between the two sextupole magnets is $n\pi/3$, one of the branches of the separatrix is parallel to the Y axis in the normalized phase plane at the position of these sextupoles. Two other branches incline from Y axis for $2\pi/3$ and $4\pi/3$ radians. So the inclinations of these three branches are $\tan(\pi/2)$, $\tan(7\pi/6)$, and $\tan(11\pi/6)$.

The separatrix rotates with the phase advance from these sextupole magnets. So the inclinations of the branches are $\tan(\pi/2-\phi)$, $\tan(7\pi/6-\phi)$, $\tan(11\pi/6-\phi)$, where ϕ is the phase advance from one of the sextupole magnets to the considered point.

Generally, a synchrotron has finite values for the dispersion function and its derivative. So the location of the separatrix in the phase plane varies with the particle momentum. When the chromaticity is corrected and the separatrices for all momenta are congruent, the condition to superimpose the separatrices around the deflector for all momenta is that one of the branches of the separatrix is parallel to the shift of the separatrix with the momentum through the dispersion function (Fig.2). The equation of the condition is:

$$\frac{\eta'}{\eta}=\frac{\tan(\theta-\phi)-\alpha}{\beta} \quad \text{with} \quad \theta=\frac{4m\pm1}{6}\pi \quad (5)$$

where ϕ is the phase advance from one of the sextupoles to the deflector, α and β are the Twiss parameters at the deflector, η and η' are the dispersion function and its derivative at the deflector. In the above equation, m is an integer which satisfies the condition $\phi-\pi/2<\theta<\phi+\pi/2$. If the position of the deflector is fixed, we can superimpose the separatrix by selecting the position of two sextupole magnets according to Eq. (5).

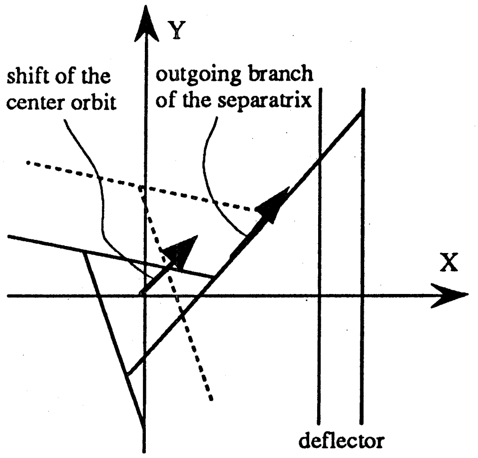

Fig.2 Separatrices with the chromaticity corrected.
The outgoing branches of the separatrices are superimposed for all momenta.

4. Application to a Compact Synchrotron

This technique has been studied for a lattice of a compact synchrotron dedicated to medical use [7]. In order to reduce the size of the synchrotron and simplify its control, this synchrotron employs a combined function lattice. The circumference is 22.9m. The bending magnets have n-indices for focusing and defocusing the beam. The horizontal tune is 1.75 and the vertical tune is 0.85. In the extraction period, two trim quadrupole magnets change the horizontal tune to nearly 1.67. The third-order resonant extraction is raised by a transverse perturbation of the radio frequency and the separatrix is kept constant. The effects of the present technique are analyzed through computer simulations.

The two sextupole magnets are treated as thin lenses. The momentum spread is assumed as $\Delta p/p=\pm0.2\%$. Emittance of the beam is assumed as 24πmm·mrad. The positions of the two sextupole magnets are selected according to Eq. (5). Horizontal tune is selected as 1.67, nearly 10π/3, in order to get proper turn separation; here the turn separation is defined as an increment of the betatron amplitude at the deflector position. Twiss parameters at the deflector and sextupole magnets are shown in Table 1.

Table. 1 Twiss parameters

Twiss parameters	deflector	sextupole magnet
β	2.96 m	1.42 m
α	-1.55	0.83
η	1.55 m	1.48 m
η'	0.39	-0.32

The strengths of the two sextupole magnets are $-0.95(m^{-2})$ and $12.05(m^{-2})$. These are selected so that the chromaticity of the synchrotron vanishe and the separatrix size becomes 24πmm·mrad.

Results of the simulation show that the separatrices for all momenta are almost congruent and the trajectories of all extracted particles are superimposed (Fig.3).

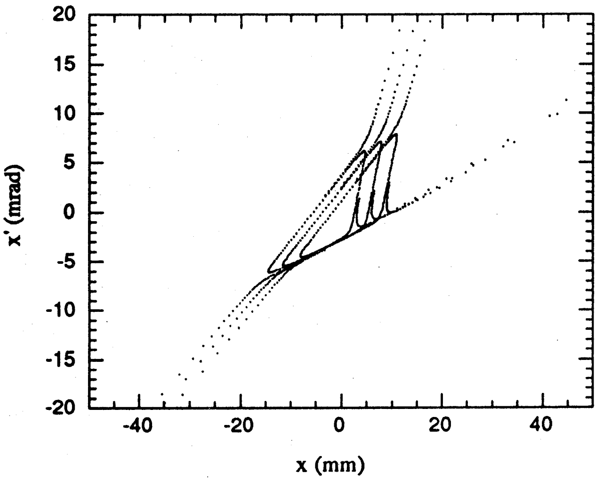

Fig.3 Results of the computer simulations.
Emittances are 24πmm·mrad for all particles of $\Delta p/p=\pm0.2\%$

Values of the turn separation at the deflector are listed in Table 2. The particles are extracted by increasing the amplitude of the betatron oscillation through the perturbation. Since the separatrices are constant during the extraction, extracted beam parameters such as the turn separations and orbit gradients, do not change.

Table. 2 Turn separation at 30mm

$\Delta p/p$ (%)	-0.2	0	+0.2
turn separation (mm)	9.9	8.3	6.6

5. Conclusion

A technique for third-order resonant extraction was given. Two sextupole magnets were drived asymmetrically, and served the two purposes, one of which is to excite the third-order resonance and the other is to correct for chromaticity. The positions of these two sextupole magnets are decided so that the outgoing branches of the separatrices at the deflector are superimposed for all momenta. This technique was applied in a compact synchrotron. Computer simulations showed that the separatrices for all momenta were almost congruent and that the trajectories of all extracted particles were superimposed. High extraction efficiency and low extracted beam emittance would be expected.

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