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LOW-EMITTANCE SLOW EXTRACTION USING HALF-INTEGER RESONANCE

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I INTRODUCTION

At the KEK-PS a resonant slow extraction using a half-integer resonance ($2\nu_h=15$) has been in operation. This article deals with a particular problem concerned with the separatrix dependency on chromaticity of resonant extraction. That dependency was first calculated by H. Bruck et al. [1] and later by W. Hardt [2] for 1/3-integer resonant extraction. They showed that the outgoing separatrices of different momentum overlap under a certain condition (overlap condition). It is desireble to obtain a separatrix branch with low divergence at the first septum (usually ESS) position in order to minimize particle losses on it. Although the design of a resonant slow-extraction system is usually done by numerical ray-tracing, analytical guide formulae for a simple estimation of the parameters is desirable. Unfortunately, the overlap condition has never been calculated for half-integer resonant extraction. This seems to be due to a difficulty in treating the parabolic form of the separatrix arm, to a straight separatrix arm of the third-integer resonance. In addition, twiss parameters (α, β and γ) introduced by Courant and Snyder [3], and the chromaticity go to infinity when the tune approaches to a half-integer. The formulae given in this article have been applied to parameter setting of resonant slow extraction at the KEK-PS.

In half-integer resonant extraction at the KEK-PS, one perturbing quadrupole magnet is inserted into the lattice in order to produce a half-integer stop-band. The total tune (ν) is quite close to a half-integer ($2\nu \approx n$). One octupole magnet is excited in order to separate phase space into stable and unstable regions. The tune is then approached slowly to a half-integer value, and the stable region becomes smaller and smaller as the tune approaches this half-integer, and the particles are ejected from the machine. This is the process of extraction.

Although particles with different momenta are usually extracted at the same time, they follow different separatrices in phase space. The difference in the angle at the ESS is the dispersion angle of the extracted beam, which is responsible for the angle divergence of the beam. One reason for the displacement is a difference in their tune, which produces a difference in the stable area in the separatrix. Another reason is a displacement by the dispersion function. These effects are schematically shown in Figure 1. Although the first displacement depends on the chromaticity and the second one does not. Therefore, when one choose the chromaticity properly, the separatrix arms of different momenta overlap as shown in Figure 1. Actually in half-integral resonant extraction, the separatrix arms do not overlap, but only cross, because the separatrix arms are not straight lines. However, this is practically sufficient for efficient slow extraction.

III CALCULATION OF SEPARATRIX

Here, the twiss parameters and chromaticity (ξ) of the unperturbed lattice are assumed to be known. Since un unperturbed lattice has no perturbation quadrupole, it also has no half-integer stop-band. Using the formula calculated by T. Suzuki and S. Kamada [4], the separatrix line in the unperturbed normalized phase space is expressed as

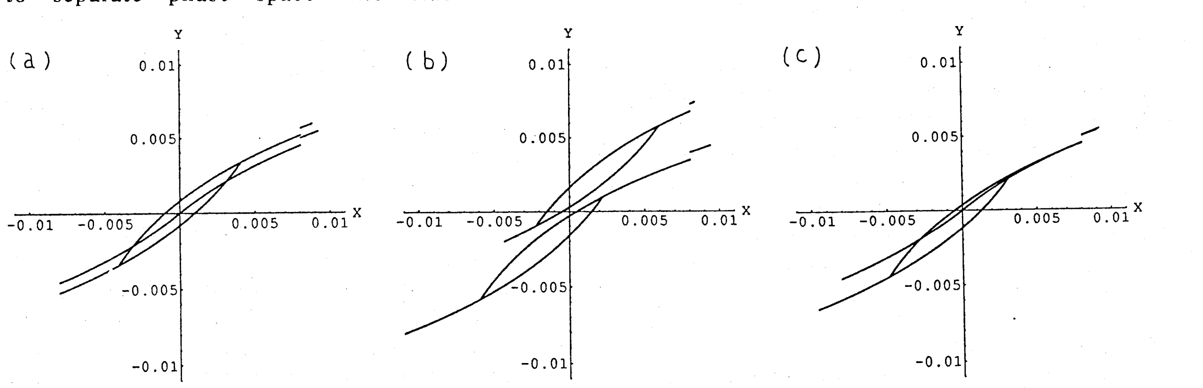

Figure 1 Separatricies in unperturbed normalized phase space. The machine parameters of the KEK-PS are used. (a) Separatricies with different emittance: 0 and 3π mm mrad. (b) Displacement by the dispersion (momentum displacement was +0.4% and -0.4%). (c) The outgoing separatrix lines are crossing at the ESS edge. One separatrix is of a particle with zero emittance and zero momentum displacement. Another separatrix is of a particle with 3π mm mrad emittance and -0.27% momentum displacement.

$$-X \sin\mu_i + Y \cos\mu_i = \tan(\mu_j-\mu_i)/\{1+(\varepsilon B)/[B\cos^2(\mu_j-\mu_i)]\}[X\cos\mu_i+Y\sin\mu_i] \pm\sqrt{D/2[B\cos^2(\mu_j-\mu_i)+(\varepsilon-B)]}\ [(X\cos\mu_i+Y\sin\mu_i)^2 -\varepsilon(\varepsilon-B)/\{D[B\cos^2(\mu_j-\mu_i)+(\varepsilon-B)]\}]. \quad (1)$$

Coordinate (X,Y) is defined according to a well-known form:

$$X \equiv x/\sqrt{\beta} \quad \text{and}$$
$$Y \equiv \sqrt{\beta}\ [x' + (\beta/\alpha)x]. \quad (2)$$

The two phases (μ_i and μ_j) are the unperturbed betatron phase advance from the entrance of the ESS to the octupole magnet and to the perturbation quadrupole magnet, respectively. The strength of these perturbations are represented by D and B, which are defined by

$$D \equiv (1/12\pi)(\Delta Bz'''\ell/B\rho)\beta_i^2 \quad \text{and} \quad (3)$$
$$B \equiv (1/2\pi)(\Delta Bz'\ell/B\rho)\beta_j. \quad (4)$$

Since the unperturbed tune (ν) has almost a half-integral value (n/2), $\varepsilon \equiv (n/2)-\nu$ is very small.

We treat the case when the perturbed tune of a particle with zero momentum displacement ($dP/P \equiv \delta=0$) is just n/2, which means that the separatrix has just zero stable area. However, a particle with $\delta \neq 0$ has a finite stable area (πE_{max}) because of the chromatic tune shift. We first calculate the separatrix when $\delta=0$. In this case $\varepsilon = B$ and the separatrix is expressed as

$$-X\sin\mu_j+Y\cos\mu_j = \pm\sqrt{D/2B}\ (X\cos\mu_i+Y\sin\mu_i)^2. \quad (5)$$

On the other hand, when $\delta \neq 0$, ε is written as $\varepsilon = B-\xi\delta$ using unperturbed chromaticity. Therefore, the separatrix is

$$-X \sin\mu_i + Y \cos\mu_i = \tan(\mu_j-\mu_i)/\{1-\xi\delta)/[B\cos^2(\mu_j-\mu_i)]\}[X\cos\mu_i+Y\sin\mu_i] \pm\sqrt{D/2[B\cos^2(\mu_j-\mu_i)-\xi\delta]}\ \{(X\cos\mu_i+Y\sin\mu_i)^2 +\xi\delta(B-\xi\delta)/\{D[B\cos^2(\mu_j-\mu_i)-\xi\delta]\}\}. \quad (6)$$

The displacement due to the dispersion function (η and η') is taken into the calculation due to the dispersion in the unperturbed normalized phase space (H and H'). Since equilibrium orbit is displaced by ($H\delta$, $H'\delta$), the X and Y of equation (6) should be replaced by $X-H\delta$ and $Y-H'\delta$, respectively. We now define the Ys so that the (X_s, Y_s) satisfy equation (5) and (X_s, $Y_s+h'\delta+h''\delta^2+$ - -) satisfy equation (6). Here, h' is the dispersion angle of the extracted beam in normalized phase space. Upon subtracting equation (5), and omitting more than the 2-nd order terms to δ, we obtain

$$H\sin\mu_j+(h'-H')\cos\mu_j = \{\xi/[2B\cos^2(\mu_j-\mu_i)]\}[X_s\sin(\mu_j-2\mu_i)+Y_s\cos(\mu_j-2\mu_i)] \pm\sqrt{2D/B}\ [X_s\cos\mu_i+Y_s\sin\mu_i][-H\cos\mu_i+(h'-H')\sin\mu_i] \pm\sqrt{B/(2D)}\ \xi/[B\cos^2(\mu_j-\mu_i)]. \quad (7)$$

To simplify the equations, we rewrite them using

$$X^* \equiv \pm\sqrt{2D/B}\ X_s, \qquad Y^* \equiv \pm\sqrt{2D/B}\ Y_s,$$
$$H^* \equiv \pm\sqrt{2D/B}\ H, \qquad H'^* \equiv \pm\sqrt{2D/B}\ H',$$
$$h'^* \equiv \pm\sqrt{2D/B}\ h' \quad \text{and} \quad (8)$$
$$\xi^* \equiv \xi/[B\cos^2(\mu_j-\mu_i)], \quad (9)$$

so that they contain neither B or D explicitly. Equations (5) and (7) become

$$2(-X^*\sin\mu_j+Y^*\cos\mu_j) = (X^*\cos\mu_i+Y^*\sin\mu_i)^2, \quad (10)$$
$$H^*\sin\mu_j+(h'^*-H'^*)\cos\mu_j +(X^*\cos\mu_i+Y^*\sin\mu_i)[H^*\cos\mu_i+(H'^*-h'^*)\sin\mu_i] = \xi^*\{1+[X^*\sin(\mu_j-2\mu_i)+Y^*\cos(\mu_j-2\mu_i)]/2\}. \quad (11)$$

The sign of equations (8) is selected in order that (X_s, Y_s) is on the outgoing separatrix arm, the step-size of X* times X* should be positive. Only one of two solutions of equation (10) is correct; that is

$$Y^*\sin^2\mu_i = F\cos\mu_j - X^*\sin\mu_i\cos\mu_i \quad (12)$$

here

$$F \equiv 1-[1-2X^*\sin\mu_i\cos(\mu_j-\mu_i)/\cos^2\mu_j]^{1/2}. \quad (13)$$

Equation (11) is solved a straightforward manner using equation (12) as

$$(h'^*-H'^*)(1-F)\cos\mu_j+H^*[\sin\mu_j+F\cos\mu_j\cot\mu_i] = \xi^*\{1+[F\cos\mu_j\cos(\mu_j-2\mu_i)/\sin^2\mu_i -X^*\cos(\mu_j-\mu_i)/\sin\mu_i]/2\}. \quad (14)$$

As has been explained, since the separatricies cross when $h'^*=0$, setting h'^* to 0 gives the crossing condition of the separatricies at the ESS.

$$\xi^*\{1+[F\cos\mu_j\cos(\mu_j-2\mu_i)/\sin^2\mu_i -X^*\cos(\mu_j-\mu_i)/\sin\mu_i]/2\} = (H^*\sin\mu_j-H'^*\cos\mu_j) +F[\cos\mu_j/\sin\mu_i][H^*\cos\mu_i+H'^*\sin\mu_i] \quad (15)$$

This is one goal of this article. However, in the calculations some small effects are ignored, such as the momentum dependence of μ_i, μ_j, α, β, γ, η, η', D and B, and the change of ξ, η and η' due to the perturbation quadrupole, and a higher order effect to δ. Although it is possible to use them in the calculation, the resultant equation is too complicated. A computer-tracking simulation would be more suitable to obtain more accurate solution. It would be helpful if we have a more simple, but less accurate, formula for the crossing condition. In the limit $X^*=0$, which means that all separatricies pass the equilibrium center, equation (15) becomes much more simple as

$$\xi^* = H^*\sin\mu_j - H'^*\cos\mu_j \quad (16)$$

We next calculate the chromaticity dependence of the angle divergence. The momentum spread of the extracted beam at one instance (δ_{max}) is determined by the emittance and chromaticity. The angle divergence at the ESS edge (ΔY in normalized phase space) is the product δ_{max} h'. Since ΔY is not constant during the extraction, we calculate the spread at the middle, where the ΔY is maximum. The δ_{max} is determined from the emittance (πE_{max}, maximum Courant Snyder invariant), which is

$$\pi E_{max}=(4\sqrt{2}/3)[\varepsilon(\varepsilon-B)]^{3/2}/\{D[B\cos^2(\mu_j-\mu_i)+(\varepsilon B)]^2\}$$
$$\approx (4\sqrt{2}/3)[|\xi^*|\delta_{max}]^{3/2}\ [B/D|\cos^2(\mu_j-\mu_i)|]\ . \quad (17)$$

Therefore,

$$\delta_{max}\approx[(3/4\sqrt{2})(\pi E_{max}D/B)|\cos(\mu_j-\mu_i)|]^{2/3}/|\xi^*| \quad (18)$$

However, when the right side of the above equation is larger than the full momentum spread of the circulating beam ($[\Delta P/P]_{max}$), δ_{max} is equal to $[\Delta P/P]_{max}$. When δ_{max} is smaller than $[\Delta P/P]_{max}$, the momentum center of the extracted beam shifts from the beginning of the extraction to the end (when the circulating beam is not accelerated during slow extraction).

III APPLICATION TO THE KEK-PS

The parameters of the KEK-PS are:

α = 2.20;	β = 15.8m at the ESS
μ_j = $(105/28)\pi$;	B = 0.051
μ_i = $-(75/28)\pi$;	D = 100/m
x_s = 32mm	
η = 1.86 m	η' = -0.1076 mrad.

Inserting these parameters into equation (15) gives ξ^*=24, i.e. ξ=0.88. This means that the angle divergence is zero when the chromaticity is set at the small positive value: 0.88. When we use the simple equation (16) the result is ξ=0.41.

The maximum angle divergence $\Delta Y_{max}/\sqrt{\beta}$ is calculated using the parameters of the KEK-PS at the top-energy: $\pi E_{max} = 3\pi$ mm.mrad and $[\Delta P/P]_{max}$ = 0.8%. The result is shown in Figure 2.

The chromaticity dependence of the extracted beam size was observed at the KEK-PS while extracting 3.5GeV protons. The extracted beam profile at several timings during the slow extraction was measured by the SWIC (Segmented Wire Ionization Chamber) at the down stream of the last magnetic septum (betatron phase advance from the ESS was about $(3/4)\pi$.) The horizontal beam profiles are shown in Figure 3. Although the beam width was smaller at a small positive chromaticity, the measurement missed the chromaticity which would have given the minimum. The shifts in the beam position from the start of extraction to the end was a reflection of the shift in the central momentum of the extracted beam. When the chromaticity was zero, the beam was split at the start of extraction because of the non-linear chromaticity. This curious beam behavior was reproduced in a computer-tracking simulation.

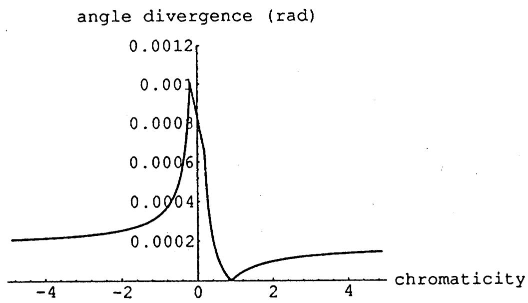

Figure 2 Maximum angle divergence at the ESS as a function of the horizontal chromaticity (ξ).

IV CONCLUSION

The crossing condition of the separatrix arms of the half-integral resonance was analytically calculated.

This has already been utilized at the KEK-PS during the extraction of protons and a light-ion beam with smaller rigidity. Unfortunately, at the time of top-rigidity the power supply of the chromaticity control sextupoles is not powerful enough to over-cancel the negative chromaticity to positive. However, below 10GeV (proton) this condition can be realized, and produced a dramatic improvement in the extraction efficiency and emittance of the extracted beam.

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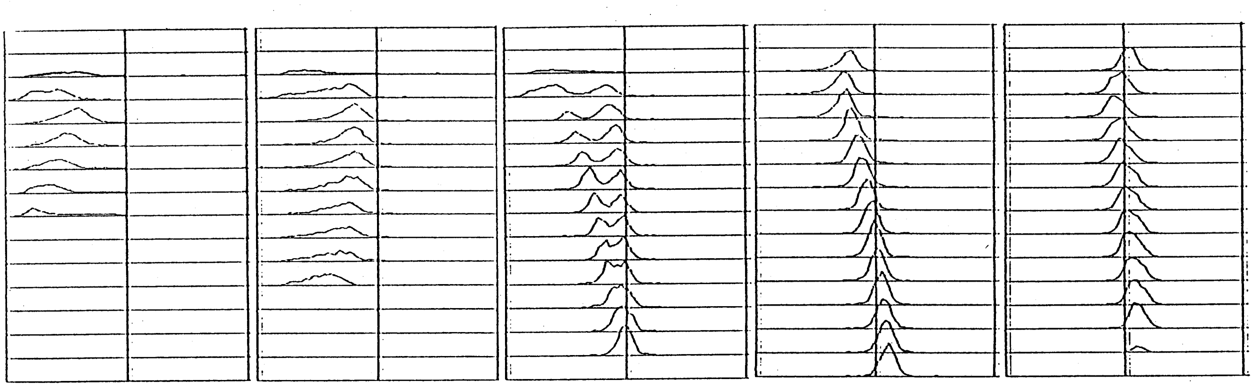

Figure 3 Horizontal beam profile of the extracted beam at the SWIC. The chromaticity was -7, -3.5, 0, +3.5, +7 from the left. The time from the top trace to the bottom trace was about 2 seconds. The full-scale of the horizontal axis is about 80mm.