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NEW CONCEPTS FOR THE SIMULATION OF BEAMS IN CYCLOTRONS

Mitsuhiro FUKUDA and Stefan ADAM*

Japan Atomic Energy Research Institute
1233 Watanuki, Takasaki, Gunma 370-12, Japan
*Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland

Abstract

New concepts have been introduced in a code for the simulation of particles in a cyclotron using transfer matrices. For the simulation the cyclotron is split into several sectors, and the calculation of the particle motion is based on tables of equilibrium orbit data at the sector boundaries, completed by tables describing the properties of the betatron oscillations. For the betatron oscillation part the new method uses three parameters that represent the focusing strengths of three thin lenses, at the start, in the middle and at the end of each sector. The new scheme has proven to be superior to the old approach based on tables of transfer matrices.

1 Introduction

A numerical integration code is normally used for calculating particle motion in a cyclotron. The particle motion in transverse and longitudinal phase space can be simulated precisely by integrating equations of motion step by step. Huge processing time is required for simulating beams consisting of many particles which have different initial conditions. The use of transfer matrices for the simulation of beams can reduce the computing time drastically. Various cyclotron orbit codes [1] [2] [3] using transfer matrices have been developed at different laboratories.

The simulation code called MATADOR [4] using first order transfer matrices has been developed to study the beam behaviour in the cyclotrons of the PSI high intensity accelerator facility. The program MATADOR has shown its merits on various problems, however, modifications of the geometric layout or an adaptation of the simulation code to other cyclotrons appeared to be cumbersome. Usually, the tables of transfer matrices produced by the orbit integration program could not be used directly in the simulation. A laborious procedure of manual smoothing of these data was often needed. Otherwise the simulation, introducing additional errors from interpolation, would show unrealistic wild oscillations of the particles.

The new representation of the transfer matrices is based on three independent parameters which are smoother functions than the elements of the transfer matrices themselves. As an additional advantage it fulfills the symplectic condition of the particle motion stating that the determinant of a transfer matrix must be exactly one. Using this new approach should give the modified simulation code the high flexibility and portability that were missing.

2 The Transfer Matrices and Their Representation

In the transfer matrix approach the motion of each particle is split up into a part describing the motion of the equilibrium orbit of the corresponding energy and another part for the difference between the actual orbit and the equilibrium orbit. The difference part can be approximated by linear betatron oscillations represented by difference vectors in (x, p_x) and (z, p_z) phase space. The difference vectors at the end of a section can be obtained from the corresponding initial vectors by matrix multiplications:

$$\begin{pmatrix} x_1 \\ p_{x1} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}_x \begin{pmatrix} x_0 \\ p_{x0} \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} z_1 \\ p_{z1} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}_z \begin{pmatrix} z_0 \\ p_{z0} \end{pmatrix} \quad (2)$$

To find the first order transfer matrices R_x and R_z and the equilibrium orbit data the orbit integration program has to calculate a central orbit and a pair of neighboring orbits for a sufficient number of energies.

According to Liouville's theorem the value of the determinant of a transfer matrix must be precisely one. Taking into account this law ($R_{11}R_{22} - R_{12}R_{21} = 1$) one of the matrix elements becomes redundant. We have therefore searched for a method to define a general two-by-two matrix using three independent parameters. In the formulation we have found the three parameters correspond to values with a physical meaning. The formula that defines the transfer matrix based on three free parameters is given by:

$$\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -F_3 & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{D}{2} \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ -F_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{D}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -F_1 & 1 \end{pmatrix}, \quad (3)$$

where F_1, F_2 and F_3 are the focusing strengths of three thin lenses, assumed to be located at the start, in the center and at the end of each section, while $D/2$ is the length of the two drift regions in between the lenses. The drift distance D is given by the spanning angle of the transfer section and the radius of the

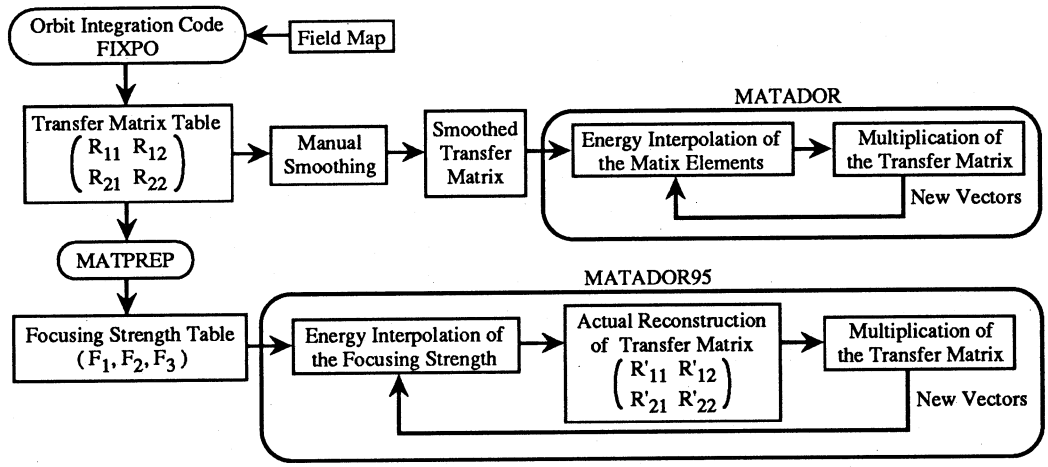

Figure 1: Comparison of the two versions of the beam simulation system. In the former procedure the transfer matrix elements for arbitrary energies were interpolated directly from the transfer matrix tables, which originated from the orbit integration code called FIXPO, but which in many cases required a manual smoothing procedure in between. The new scheme conveys the transfer matrix information as tables of focusing strengths for thin lenses created in the data preparation program MATPREP. These sets of focusing strength data are used for the energy interpolation in the modified program called MATADOR95. In MATADOR95 the reconstructed transfer matrices are then based on interpolated values of the focusing strength parameters.

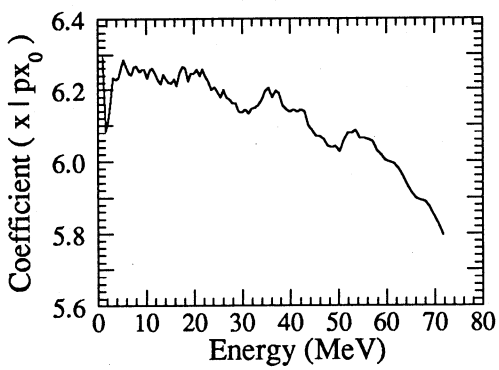

Figure 2: Energy dependence of the transfer matrix element R_{12} for the transfer section from one of the acceleration gaps to the center of the following sector magnet in the PSI Injector II, derived from magnetic field data. The strong oscillations seen in this function pose problems for its interpolation.

equilibrium orbit. The focusing strength is defined reciprocal to a conventional focal length in order to avoid an indefinite condition when focusing is zero. In this formulation the determinant of the transfer matrix automatically remains unity.

The thin lens system can be applied to any transfer section, since zero focusing strength gives a unit matrix for the corresponding matrix factor. For example, if a transfer section involves zero focusing for all three lenses, the resulting matrix from the equation (3) corresponds to the transfer matrix for a drift section of length D.

3 Implementing the New Parameterization of Transfer Matrices

A flow chart underlining the differences between the two representations of the transfer matrices is shown in Fig. 1. A transfer matrix table is created in the orbit integration code called FIXPO. The table contains a large number of parameter sets to cover all the transfer sections and a number of energies corresponding to the number of turns of an accelerated beam.

Because these raw data have to be interpolated, a time consuming manual smoothing procedure is required in most cases to make them useable for the simulation program. Figure 2, as an example, shows the energy dependence of the element R_{12}, which is the coefficient of $(x|p_{x0})$, in the transfer matrix for the transfer section from one of the acceleration gaps to the center of a sector magnet in the PSI Injector II cyclotron. The strong fluctuations shown in this figure make it clear that the interpolation of such a function can pose problems. As an additional disadvantage of the old scheme, the property of the transfer matrix, that the determinant is equal to one, gets lost due to the interpolation process.

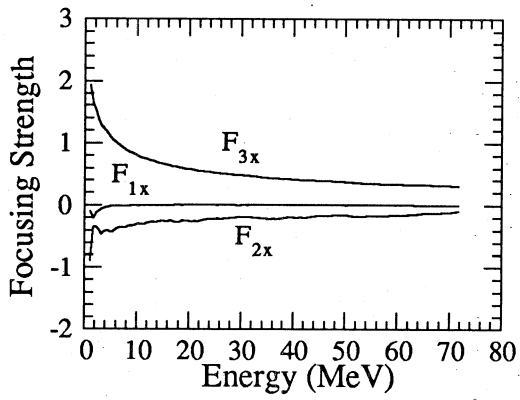

Figure 3: Energy dependence of the horizontal focusing strength for the same transfer section of the Injector II as in Fig. 2. The lines labelled F_{1x}, F_{2x} and F_{3x} represent the focusing strength at the start, in the center and at the end of the transfer section, respectively.

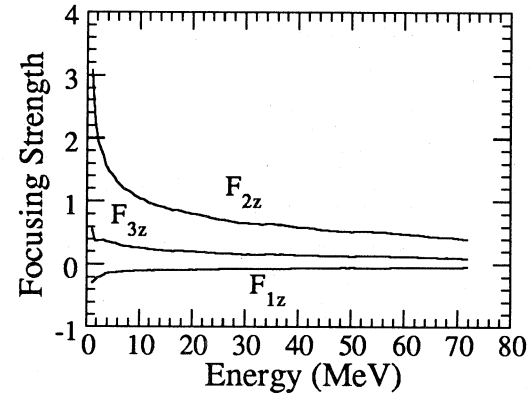

Figure 4: Energy dependence of the vertical focusing strength for the same transfer section of the Injector II as in Fig. 2. The lines labelled F_{1z}, F_{2z} and F_{3z} represent the focusing strength at the start, in the center and at the end of the transfer section, respectively.

In the new scheme the program MATPREP prepares tables of focusing strength parameters based on the raw transfer matrix tables. The energy dependence of these focusing strength parameters, for the same transfer section as in Fig. 2, is shown in Fig. 3 and Fig. 4. The energy dependence is much smoother than in Fig. 2. Thus, the errors in the energy interpolation of the focusing strength can be reduced in comparison to the case where the matrix element is interpolated itself and no need occurs for a manual smoothing procedure. In the new scheme the determinant of all interpolated transfer matrices now remains exactly one. In the modified code of MATADOR95 the sets of focusing strength parameters are interpolated, and after the interpolation the transfer matrix is reconstructed by using the equation (3).

The physical meaning of the focusing strength parameters is illustrated in Fig. 3 for the horizontal and in Fig. 4 for the vertical focusing. In both cases the first parameter (F_{1x} or F_{1z}) is almost zero, as its azimuth is located in the field free region. For the second and third focusing strength parameters the x-direction and the z-direction show different characteristics. The third parameter shows a strong focusing in the x-direction explained by the fact that it is located on the magnet centerline. For the z-parameters, the second one, F_{2z}, is the largest due to the vertical focusing produced by the angle between the orbit and the magnet edge.

4 Conclusions

The new concept that represents beam transfer matrices based on three free parameters has been successfully implemented in the particle simulation program MATADOR95 at PSI. Results from test simulations are similar to the ones from the earlier program, but the new program can be considered to be more reliable due to its guarantee that the determinant of a transfer matrix equals one. The fact that the focusing strength parameters are naturally smoother functions than transfer matrix elements avoids the need for a manual processing of the results from the orbit integration program. This provides more flexibility to study variations of the accelerator geometry and easily enables the use of the new code for arbitrary cyclotrons.

5 References

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