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STUDY OF INCOHERENT BEAM - BEAM EFFECTS AT RADIOACTIVE ISOTOPE BEAM FACTORY

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Abstract

Merging ion-ion collisions is an important feature of the proposed RIKEN Radioactive Isotope Beam Factory[1]. In the merging collision case, the value of luminosity 10^{26} 1/cm^{2}sec is several orders of magnitude less than for head-on collisions because both beams have almost the same vector of velocity and merging angle is rather small (1-10^{o}) even when the stored number of ions is close to the space charge limit of 10^{12} particles. In the present paper, the beam-beam effects are studied for the coasting merging beam collisions using particle-in-cell (PIC) model in 4D phase space. Tolerable incoherent beam - beam tune shift and beam disruption effect such as emittance growth have been evaluated from high order nonlinear resonances study. Beam luminosity and beam life time due to beam-beam effects are estimated as a function of main collider parameters.

1. Introduction

Proposed Radioactive Isotope Beam Factory is aimed to be used for wide range of experiments with unstable nuclear beams. Among variety of planning experiments the ion-ion merging collisions are of the most importance. Merged beam technique is very useful method for the study of nuclear fusion processes. Merging two RI beams deliver low energy collisions just above the Coulomb barrier threshold that is difficult to be realized in other experimental methods. The most important collider parameter is luminosity which is limited among other reasons by physics of beam-beam interaction. In this paper we analyze the luminosity constraints originating from a beam-beam interaction as a function of main parameters of storage ring.

2. Luminosity of merge beam - beam interaction

We consider two coasting merge ion beams with particle densities n_1, n_2 and beam velocities $\vec{v}_1 = c\vec{\beta}_1$, $\vec{v}_2 = c\vec{\beta}_2$ colliding with angle α (see fig. 1). Luminosity L is defined as a ratio of interaction rate to cross section for particle interaction $L = 1/\sigma \, dN/dt$. Using expression for invariant cross section[2] the number of collisions dN during the time dt is

$$\frac{dN}{dt} = \int_V \sigma \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - \frac{[\vec{v}_1 \times \vec{v}_2]^2}{c^2}} \, n_1 n_2 \, dV \qquad (1)$$

where integration is performed over the volume of interaction. It is convenient to express luminosity as a

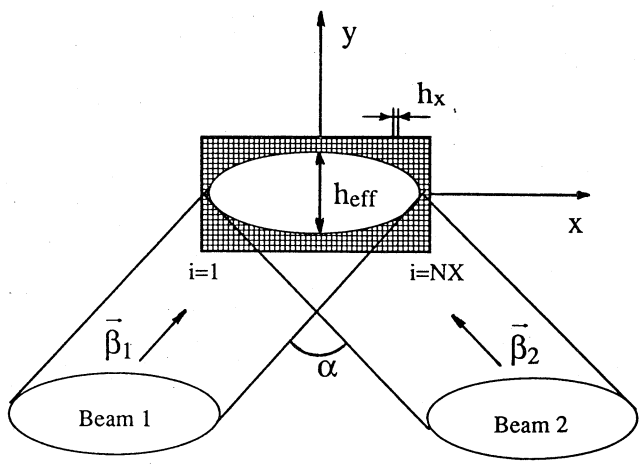

Fig.1 Merge beam - beam interaction.

function of collision angle α, number of particles per beam N_1, N_2, ring circumference $2\pi R$ and effective size of the beam h_{eff} at the interaction point:

$$L = \frac{\sqrt{\beta_1^2 + \beta_2^2 - 2\beta_1\beta_2\cos\alpha - \beta_1^2\beta_2^2\sin^2\alpha} \; c \, N_1 N_2}{(2\pi R)^2 \sin\alpha \; h_{eff}} \qquad (2)$$

Let us note that for merge coasting beams the luminosity is inversely proportional to beam height and does not depend on beam width.

3. Particle-in-Cell Model of Beam-Beam Interaction

Beam-beam interaction was studied by combination of particle-in-cell treatment of space charge problem at the crossing point and transfer matrix for particle revolution in storage ring. Self - consistent consideration of the problem is based on calculation of beam-beam interaction arising from the intrinsic space charge field of the beams. Beams are represented as a collection of large number (10^3 - 10^4) modeling particles. Real number of particles in the beams is much larger (10^{12}). To reduce the numerical fluctuation arisen from over evaluated particle-particle collisions, the smoothing particle-in cell technique is used. For merging ion-ion collision we use the assumption that two beams are identical therefore we can consider behavior of only one beam (the second beam is the same). At the point of interaction influence of one beam on another one is simulated as strong-strong interaction of two identical beams with crossing angle.

Simulation starts with random number generation of initial particle positions x,y and reduced particle momentum $p_x = \beta_x\gamma$, $p_y = \beta_y\gamma$ in four-dimensional phase space. Initial distribution of the beam in transversal phase space coordinates gives the elliptical phase space projections described by root-mean-square (RMS) beam ellipse $a_o x^2 + 2b_o x p_x + c_o p_x^2 = \varepsilon$ where the RMS beam emittance ε is defined as

$$\varepsilon = 4\sqrt{<x^2><p_x^2> - <xp_x>^2} \qquad (3)$$

analogously for y, p_y. Simulation of each turn of particle with longitudinal momentum $p_s = \beta_s\gamma$ consists of transfer mapping of one revolution of particle in storage ring of radius R with betatron tunes Q_x, Q_y and nonlinear kick Δp_x, Δp_y due to beam-beam interaction:

$$x^{n+1} = x^n \ (\cos 2\pi Q_x) \ + p_x^n \ \left(\frac{R}{p_s} \frac{\sin 2\pi Q_x}{Q_x}\right) \qquad (4)$$

$$p_x^{n+1} = x^n \left(-\frac{p_s Q_x}{R} \sin 2\pi Q_x\right) + p_x^n \ (\cos 2\pi Q_x) + \Delta p_x \qquad (5)$$

analogous for y^{n+1}, p_y^{n+1}. Self-consistent beam-beam kicks Δp_x, Δp_y are calculated from space charge problem with instantaneous distribution of particles :

$$\Delta p_x = \frac{q}{mc^2} \frac{h_x}{tg\alpha\beta_s\gamma^2} \sum_{i=1}^{NX} E_{x\,i}(y) \qquad (6)$$

$$\Delta p_y = \frac{q}{mc^2} \frac{h_x}{\sin\alpha\beta_s\gamma^2} \sum_{i=1}^{NX} E_{y\,i}(y) \qquad (7)$$

where mc^2/q is a rest energy divided by charge of particle, E_{xi} (y), E_{yi} (y) are space charge field of the beam calculated at spatial grid points, NX is a number of equidistant mesh points along x - axis located with step h_x (see fig.1). Space charge field of the beam is calculated from the Poisson's equation in Cartesian coordinates

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -\frac{\rho(x,y)}{\varepsilon_o} \qquad (8)$$

with Dirichlet boundary conditions for potential U on the surface of conducting pipe using Fast Fourier Transforms.

4. Beam-Beam Tune Shift

In coasting merge beams the interaction is not x-y symmetric. In the median plane of the beams (x-direction in fig. 1) particles experience much smaller kick than in the vertical plane (y-direction). As a result, initially round beams become prolonged in the direction perpendicular to median plane. Realistic beam-beam kick is a nonlinear function of transverse coordinate due to nonlinear space charge field of the beam. Gaussian beam with N particles and transverse RMS size σ delivers space charge field:

$$E_r = \frac{Nq}{4\pi^2 R\,\varepsilon_o} \frac{1}{r} [1 - \exp(-\frac{r^2}{2\sigma^2})] = \frac{Nq}{8\pi^2 R\varepsilon_o\sigma^2} r + ... \qquad (9)$$

From eqs. (7), (9) the linear approximation to beam-beam kick is as follows:

$$\Delta p_y = \frac{N q^2}{2\pi^2 mc^2\varepsilon_o \beta\gamma^2 \sin\alpha \ R\,\sigma_y} \ y \qquad (10)$$

Multiplication of matrix of one turn and linear beam-beam kick gives the value of linear betatron tune shift for merge beam-beam collisions:

$$\xi_y = \frac{N r_o}{2\,\pi^2\sigma_y \sin\alpha \ \beta^2 \gamma^3 Q_y} \qquad (11)$$

where $r_o = q^2/ 4\pi\varepsilon_o mc^2$ being the value of classical radius of particle.

5. Results of simulation

RI beam factory is supposed to have 2 colliding points, therefore the values of tune are half of betatron tunes in the ring: $Q_x/2 = 2.8815$; $Q_y/2 = 3.175$. For two coasting merge beams (see fig. 1) only y - direction is responsible for degradation of beam luminosity due to compensation of beam-beam kick in x-direction. The closest resonance value $mQ_y = n$ to betatron tune value 3.175 is $6Q_y = 19$, i.e. 6th order resonance is achieved for $Q_y = 19/6 = 3.16666$. Max tune shift from working point to resonance is $\xi = 3.175 - 3.1666 = 0.00833$. At fig. 2,3 and in Table the results of calculations for $\xi = 0.005$ (non-resonance case) and $\xi = 0.027$ (resonance case) are presented. Different initial nonlinear particle distributions were treated in calculations: Gaussian distribution $\rho(r) = \rho_o \exp(-r^2/2\sigma^2)$ and "parabolic" distribution $\rho(r) = \rho_o (1 - r^2/R_o^2)^2$. Evolution of RMS beam envelope

$$2\sigma_x = 2\sqrt{<x^2>} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} x_i^2} \qquad (12)$$

and RMS beam emittance ε (see eq.3) were controlled during the simulations as statistical averaged values over large number of modeling particles. From results of simulations it follows that below resonance threshold $\xi < 0.008$ beams are stable, i. e. no envelope and RMS emittance growth were observed. Under resonance conditions $\xi > 0.00833$ beam-beam instability was observed. The Gaussian distribution is more unstable than the parabolic distribution. Under the Gaussian distribution the phase space portrait of the beam is typical for high order resonance (see fig.2) while for parabolic distribution the phase space projections are ellipses as for non-resonance case .

Table. Results of PIC simulation of beam-beam effects (mesh NX x NY = 64 x 64).

Beam Distribution	Tune	Turns	Number of particles	Envelope Growth	Emittance Growth (per 10^4 turns)
Gaussian	0.005	$4\cdot10^4$	10^3	1.0 (no growth)	1.0002
Gaussian	0.027	10^4	$5\cdot10^3$	1.03	1.05
Parabolic	0.027	$2\cdot10^4$	$5\cdot10^3$	1.01	1.025

6. Limitation of Luminosity and Beam Lifetime

The calculation performed shows the limitation in max tune shift due to beam-beam effects $\xi < 0.008$ which is typical for ion-ion collisions. Assuming $\xi_{max} = 0.008$, $\sigma_y = 10^{-3}$m, $\alpha = 1^o$, $\beta\gamma = 1.7$, $r_0 = 1.5\ 10^{-18}$m (proton) the limited number of particles due to beam-beam interaction is (see eq.11):

$$N < 2\pi^2 \frac{\xi_{max}\, Q_y\, \sigma_y \sin\alpha\, \beta^2 \gamma^3}{r_o} = 6\cdot10^{13} \qquad (13)$$

This value is larger than the space charge limited number of particles in the ring of radius R=28m due to incoherent space charge tune shift (Laslett tune shift) $\Delta\nu_{max} = 0.25$:

$$N < 4\pi \frac{\Delta\nu_{max}\, Q_y\, \beta^2 \gamma^3 \sigma_x^2}{R\, r_o} = 3\cdot10^{12} \qquad (14)$$

Taking limited number of particles $N_{max} = 3\cdot10^{12}$, $\alpha = 10^o$, $h_{eff} = 4\cdot10^{-3}$m and assuming $\beta_1 = \beta_2 \approx 1$, the limitation in luminosity is (see eq. 2):

$$L < \frac{N_{max}^2\ c}{(2\pi R)^2 h_{eff}} \operatorname{tg}\frac{\alpha}{2} = 2\times10^{26} \frac{1}{\sec\ cm^2} \qquad (15)$$

From results of simulation we observe the increase of transverse beam size 1-3% per 10000 turns at the resonance conditions. If we assume that for serious degradation of luminosity the beam size should expanded twice, the upper limit of beam lifetime in resonance is

$$N_{max} = \frac{100\%}{2\%}\ 10000 = 5\times10^5 \text{ turns.} \qquad (16)$$

If the resonances are avoided, the beam life time is limited by other reasons.

7. Conclusions

Particle-in-cell simulation shows significance of beam-beam interaction on beam parameters. Beam lifetime and luminosity constraints were studied from strong-strong model of beam-beam interaction. Max number of particles estimated from beam-beam interactions exceeds the limit defined by incoherent space charge tune shift for circulated beam. Further study is required for estimation of synchro-betatron resonances on beam parameters due to finite length of the interacted bunches.

8. References

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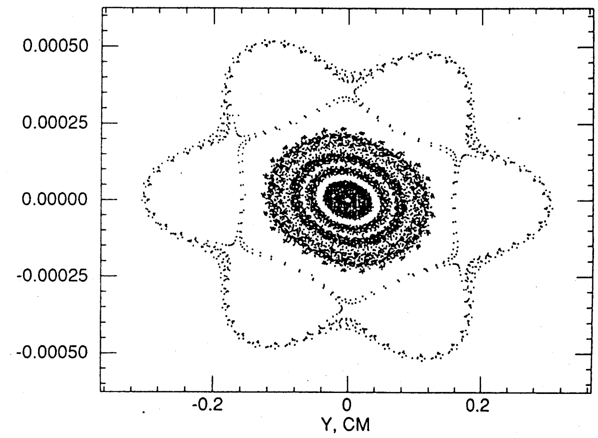

Fig.2. Phase space trajectories of Gaussian beam near nonlinear resonance of 6th order ($\xi_y = 0.027$).

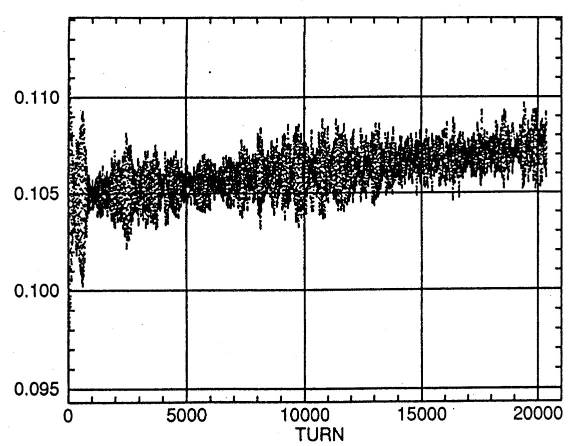

Fig. 3. Envelope growth of the beam with parabolic distribution, $\xi = 0.027$.