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# Nonlinear Resonances in a Multi-Stage Free-Electron Laser Amplifier

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## Abstract

Nonlinear resonances in the longitudinal phase-space of a multi-stage Free-Electron Laser for the Two-Beam Accelerator have been studied. We have developed a new analytic theory based on the macroparticle model and the perturbation method. A resonance-structure observed in simulations is found to be modeled by the nonlinear pendulum equation and to depend on a waveguide dimension.

## 1. Introduction

A Two-Beam Accelerator (TBA) is a possible candidate for future linear colliders, in which rf power required for a high-gradient linac is provided from a Multi-stage Free-Electron Laser (MFEL) [1,2] as shown in Fig. 1. The MFEL has some unique features. First a bunched beam drives each FEL. Second it has a periodicity: after amplification of input seed-power in each FEL, the driving beam is re-accelerated with an induction unit for energy replenishment to go to the succeeding stage. This fact means that rf and beam characters vary periodically and a bucket evolves rapidly in each FEL.

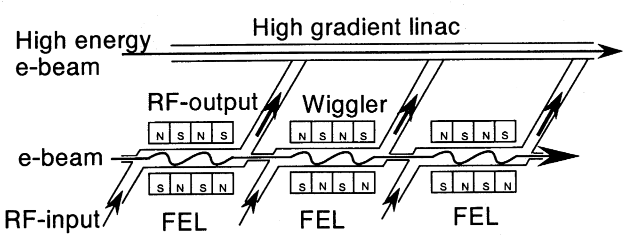


Fig. 1 Schematic picture of TBA/FEL

One of the interesting issues in the MFEL is the resonances between the synchrotron motion in a bucket and the periodicity of the MFEL [3]. The resonances lead to a formation of islands within a rf bucket in longitudinal phase-space, and degrade the performance of the MFEL as an rf source. We suppose that the resonances should be serious when the power-density of the amplified rf is strong.

This paper presents the nonlinear resonance in the MFEL for the recent version of TBA [4,5], not for the early one [3]. Section 2 briefly shows the simulation results which show the existence of the resonance. We show in section 3 that by use of the macroparticle model [6-8], the motion of electron in the phase space of the MFEL can be described by the nonlinear pendulum equation with periodically and rapidly time-varying "mass" and "length". Section 4 shows how the resonance can be theoretically analyzed with the perturbative calculation.

## 2. Simulation

The well-known one dimensional FEL simulations [9,10] have been performed. We have assumed a rectangular waveguide $TE_{01}$ mode as a signal wave. Typical parameters of the MFEL of interest are listed in Table 1. The high rf power-density with a rapidly increasing ponderomotive force would give rise to strong resonances. In order to evaluate effects of the rf power-density on the resonances, only the waveguide width $a^*$ is varied while the other parameters are fixed. For a relatively wide waveguide ($a^* \geq 10$ cm), our previous simulation shows that the beam propagates from the first to the 300-th stage without detrapping, and maintains the original bunch shape [7]. For cases of smaller waveguide $a^* = 8$, 4 cm, meanwhile, the fourth and third-integer resonances are observed as shown in Fig. 2. These resonances are considered to be caused by the strong rf power-density resulting from the reduced waveguide width. When these resonances occur, the beam continues to lose its population, the rf power decreases, and the signal phase changes [4]. These are significant problem for the multi-stage rf-source where transport of a high current beam over a long distance is indispensable to maintain a constant amount of amplified rf power.

**Table 1.** FEL parameters for TBA/FEL

| | | |
|---|---|---|
| beam current $I_e$ | 2 | kA |
| beam energy $\gamma$ | 23 | |
| energy gain per period $\Delta\gamma$ | 1 | |
| wiggler wave length $\lambda_w$ | 26 | cm |
| wiggler length per period $L_w$ | 52 | cm |
| wiggler peak field $B_w$ | 3.85 - 3.6 | kG |
| signal frequency $f_s$ | 17.1 | GHz |
| input rf power $P_{in}$ | 10 | MW |
| waveguide width $a^*$ | 20 - 4 | cm |
| waveguide height $b^*$ | 3 | cm |
| number of FEL stage | 300 | |

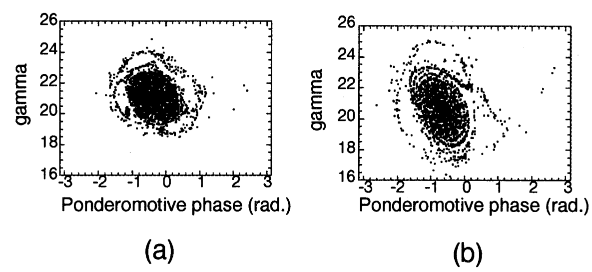


FIG. 2 Longitudinal phase-space structure by FEL simulations. (a) the fourth-integer resonance (waveguide width $a^*$ = 8 cm). (b) the third-integer resonance ($a^*$ = 4 cm).

## 3. Nonlinear pendulum equation

Following the Ref. 3 and using the definition of the macroparticle [6-8], the Hamiltonian for the MFEL can be written as

$$H(\varepsilon,\xi,E,\chi;z) \cong \sum_i^N \left\{ (k_w - \delta k_s)(\gamma_a + \varepsilon) + \frac{\omega_s(1+a_w^2)}{2c(\gamma_a+\varepsilon)} - a_w \left(\frac{eZ_0 J_e}{m_e c^2}\right)^{1/2} \frac{(E - N\gamma_a)^{1/2}}{N^{1/2}} \frac{\cos(\psi_a + \xi)}{\gamma_a + \varepsilon} + \frac{d\gamma_a}{dz}\xi \right\}, \tag{1}$$

where $\gamma_a$, $\psi_a$ are the Lorentz factor and ponderomotive phase of the macroparticle, $\varepsilon$, $\xi$ are deviations from the energy and phase of the macroparticle, $N$ the number of particles, $a_w = eB_w/\sqrt{2}m_e ck_w$ the normalized wiggler amplitude, $\delta k_s = \omega_s/c - k_s$ the shift of longitudinal wavenumber from its value in vacuum, $\omega_s$ rf angular frequency, $k_s = \sqrt{(\omega_s/c)^2 - (\pi/a^*)^2}$ the wavenumber for $TE_{01}$ mode, $J_e$ beam current density, $e_s$ normalized signal-field, $k_w$ the wavenumber of wiggler, $\varphi_s$ the signal phase, $e$ and $m_e$ are the charge and rest mass of electron, $c$ the speed of light, $z$ longitudinal coordinate, $Z_0 = 377\Omega$, $E = Nm_e c^2 e_s^2/eZ_0 J_e$ and $\chi = -\varphi_s$. Expanding the Hamiltonian (1) in powers of $\varepsilon/\gamma_a$ and retaining the dominant terms, we have

$$H(\varepsilon,\xi;z) \cong G(z)\varepsilon^2/2 - F(z)\cos\psi_a\cos\xi + F(z)\sin\psi_a(\sin\xi - \xi), \tag{2}$$

where $G(z) = \omega_s(1+a_w^2)/c\gamma_a^3$, $F(z) = a_w e_s/\gamma_a$. Neglecting friction terms which are proportional to $\xi'$ because of their smallness, we obtain a nonlinear pendulum equation

$$\xi'' + GF\{\sin\psi_a(\cos\xi - 1) + \cos\psi_a \sin\xi\} \approx 0, \tag{3}$$

from the Eq. (2), where primes denote differentiation with respect to $z$. The macroparticle model assumes $\gamma_a \propto a_w$; hence $GF$ in Eq. (3) is determined mainly by $e_s$ which is written in a term of the trigonometric function [8], $e_s(z) \cong (2\kappa/a|b|)\sin(|b|z/2)$, where $a$, $b$, $K$, and $\kappa$ are constants defined in Refs. 6, 7.

Figure 3 shows the results of numerical integration of Eq. (3) for $a^*$ = 8, 4 cm. We observe that Eq. (3) can well reproduce the results of the FEL simulations seen in Fig. 2. Thus, we consider the Hamiltonian (1) with $E \propto e_s^2$, $\chi = -\varphi_s$ determined by the macroparticle model as a theoretical base to analytically assess the nonlinear resonances in the MFEL.

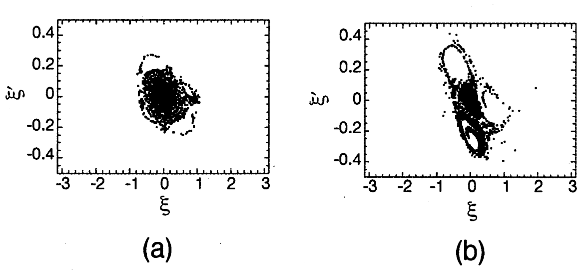


FIG. 3 Phase-space plots of the nonlinear-pendulum's solution for (a) $a^*$ = 8 cm and (b) $a^*$ = 4 cm.

## 4. Isolated resonance theory

Using the isolated resonance theory [11] which has been established in beam dynamics of circular accelerators, we can calculate the size and position of the primary resonance islands [4]. Expanding the Hamiltonian (1) in powers of $\xi$ and $\varepsilon/\gamma_a$:

$$H(\varepsilon,\xi;z) = G\varepsilon^2/2 + F_C\xi^2/2 + (perturbations) \quad (4)$$

where $F_c = F\cos\psi_a$. The expression can be regarded as the non-autonomous one degree of freedom Hamiltonian for a pendulum with time-varying "mass" $G$ and "length" $F_c$, affected by nonlinear perturbations. Using the generating functions $F_2(\xi,\bar{\varepsilon};z) = \xi\bar{\varepsilon}/\sqrt{G(z)}$ and $F_1(\bar{\xi},\theta) = -(\alpha + \tan\theta)\bar{\xi}^2/2\beta$, where $\alpha$, $\beta$ satisfy

$$2\beta\beta'' - \beta'^2 + 4GF_c\beta^2 = 4, \quad (5a)$$

$$\alpha = -\beta'/2, \quad (5b)$$

the Hamiltonian (4) is transformed to

$$H(J,\theta;z) = (\nu_0/\beta(z))J + (perturbations). \quad (6)$$

Here $\beta$ is referred to as a longitudinal amplitude function in the MFEL, which is a quantity analogous to a transverse amplitude function in circular accelerators. This function represents the orbital evolution of the bunch envelope in the MFEL. Instead of $z$, we use a new independent variable $\sigma$ defined by $\sigma = (1/\nu_0)\int_0^z 1/\beta(z')dz'$, where $\nu_0 = (1/2\pi)\int_{L_w} 1/\beta(z')dz'$ is referred to as the longitudinal tune and $2\pi\nu_0$ is the phase advance per FEL period. Retaining only the dominant terms after some straightforward mathematical manipulation and according to the isolated resonance theory [11], we have the "time"-independent Hamiltonian for the isolated third-integer resonance:

$$H_3(J,\theta) = \delta_{m/3} J + h_1 J^2 + h_3 J^{3/2}\sin(3\theta + \Theta_3), \quad (7)$$

where $\delta_{m/3}$, $h_1$, $h_3$ and $\Theta_3$ are all constants which depend on FEL parameters [4]. The Hamiltonian for the fourth-integer resonance also can be calculated in a similar way [4]. Eventually we arrive at the exact mathematical formula necessary to theoretically asses the primary resonance observed in the simulations. Fig. 4 shows lines of the equi-Hamiltonian in the phase-space for $a^*$ = 8, 4 cm in the rectangular coordinates $P = \sqrt{2J}\sin\bar{\theta}$, $Q = \sqrt{2J}\cos\bar{\theta}$ to compare with Figs. 2, 3. The calculated position and size of the islands are in agreement with simulations [4]. The transition of phase-space structures in Figs. 2, 3 also can be explained with the above theory [4]. The longitudinal tune $\nu_0$ increases with a decrease of the waveguide width $a^*$. When $\nu_0$ is more than 1/4 (1/3), the fourth (third)-integer resonance can occur.

This theory is able to give a crucial suggestion in choosing practical MFEL parameters.

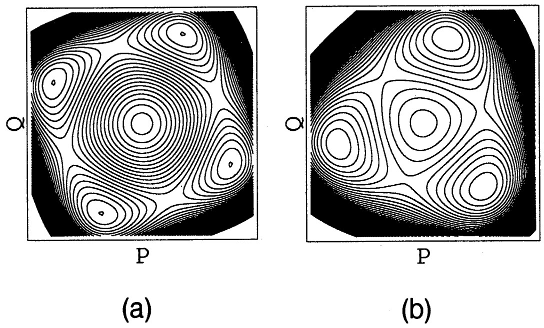


FIG. 4 Phase-space plots of the equi-Hamiltonian for (a) the fourth-integer resonance ($a^*$ = 8 cm) and (b) the third-integer resonance ($a^*$ = 4 cm).

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