Spin Depolarization in Presence of Beam-Beam Interaction

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Abstract

Analytical and numerical treatment of spin depolarization due to ion - ion collisions is given. It is shown, that in a collider with two Siberian Snakes in each ring spin depolarization due to beam-beam collision is suppressed. Degree of depolarization can be controlled by an appropriate choice of working point of betatron particles motion. In the absence of Siberian Snakes, beam-beam collision results in monotonous spin depolarization.

1. Introduction

Particle colliders with polarized beams require careful control of spin depolarization. During acceleration spin is subjected to intrinsic and imperfection resonances, resulting in depolarization. Extra source of depolarization is beam-beam collisions. In present paper effect of beam-beam collision on spin depolarization in an ion - ion collider is studied. Analytical treatment of the problem provides choice of the collider operation point, where depolarization is minimized. It also indicates zone of relatively strong depolarization.

2. Collider Model With Polarized Beams

Let us consider a collider ring with two installed Siberian Snakes. We use a two-dimensional particle model in phase space (x, p_x), (y, p_y), where x and y are particle positions, $p_x = \beta_x^*(dx/dz)$ and $p_y = \beta_y^*(dy/dz))$ are particle momentum, $\beta_x^* = R/Q_x$ and $\beta_y^* = R/Q_y$ are average values of beta-functions of the ring, R is a ring radius, Q_x and Q_y are betatron tunes. Particle motion between subsequent collisions combines linear matrix with betatron angles $\bar{\theta}_x = 2\pi Q_x$, $\bar{\theta}_y = 2\pi Q_y$, perturbed by beam-beam interaction:

$$\begin{pmatrix} x_{n+1} \\ p_x^{n+1} \\ y_{n+1} \\ p_y^{n+1} \end{pmatrix} = \begin{pmatrix} \cos\bar{\theta}_x & \sin\bar{\theta}_x & 0 & 0 \\ -\sin\bar{\theta}_x & \cos\bar{\theta}_x & 0 & 0 \\ 0 & 0 & \cos\bar{\theta}_y & \sin\bar{\theta}_y \\ 0 & 0 & -\sin\bar{\theta}_y & \cos\bar{\theta}_y \end{pmatrix} \begin{pmatrix} x_n \\ p_x^n + \Delta p_x^n \\ y_n \\ p_y^n + \Delta p_y^n \end{pmatrix}. \quad (1)$$

Beam-beam kicks Δp_x^n, Δp_y^n are expressed as a result of interaction of particles with opposite beam with Gaussian distribution function

$$\Delta p_x^n = 4\pi\xi x_n \frac{1 - \exp[-r_n^2/(2\sigma_n^2)]}{[r_n^2/(2\sigma_n^2)]}, \quad (2)$$

and similar for Δp_y^n. Parameter ξ is a beam-beam parameter, which characterizes the strength of interaction:

$$\xi = \frac{N r_o \beta^*}{4\pi\sigma^2\gamma}, \quad (3)$$

where N is a number of particles per bunch, $r_o = q^2/(4\pi\varepsilon_o mc^2)$ is a classical particle radius, σ is a transverse standard deviation of the opposite beam size and γ is a particle energy.

Rotation of spin vector $\vec{S} = (S_x, S_y, S_z)$ is described by subsequent spin matrix transformation [1, 2]. Matrix of spin advance in an lattice arc is described as a matrix of spin rotation in dipole magnet with bending angle υ:

$$D_\upsilon = \begin{vmatrix} \cos(\omega\delta s) & 0 & \sin(\omega\delta s) \\ 0 & 1 & 0 \\ -\sin(\omega\delta s) & 0 & \cos(\omega\delta s) \end{vmatrix}, \quad (4)$$

where $\omega\delta s = (1+G\gamma)\upsilon$ and G = 1.7928 is the anomalous magnetic moment of the proton. Matrixes of Siberian Snakes are given by

$$S_1 = \begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}, \quad S_2 = \begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix}. \quad (5)$$

Spin advance after crossing the interaction point is described as follow :

$$\begin{vmatrix} S_x \\ S_y \\ S_z \end{vmatrix} = \begin{vmatrix} 1-a(B^2+C^2) & ABa + Cb & ACa + Bb \\ ABa - Cb & 1-a(A^2+C^2) & BCa - Ab \\ ACa - Bb & BCa + Ab & 1-a(A^2+B^2) \end{vmatrix} \begin{vmatrix} S_{x,o} \\ S_{y,o} \\ S_{z,o} \end{vmatrix}, \quad (6)$$

$$A = \frac{P_x}{P_o}, \quad B = \frac{P_y}{P_o}, \quad C = \frac{P_z}{P_o}, \quad P_o = \sqrt{P_x^2 + P_y^2 + P_z^2}, \quad (7)$$

$$a = 1 - \cos(P_o\,\delta z), \quad b = \sin(P_o\,\delta z), \quad (8)$$

where $\delta z = l/2$ is an interaction distance and l is a bunch length. Vector $\vec{P} = (P_X, P_Y, P_Z)$ is given as follow:

$$P_X = \frac{1}{B\rho}\left[(1+G\gamma)B_X + \left(G\gamma + \frac{\gamma}{1+\gamma}\right)\frac{\beta E_Y}{c}\right], \quad (9)$$

$$P_Y = \frac{1}{B\rho}\left[(1+G\gamma)B_Y - \left(G\gamma + \frac{\gamma}{1+\gamma}\right)\frac{\beta E_X}{c}\right], \quad P_Z = 0, \quad (10)$$

where Bρ is a rigidity of particles, $\vec{E}=(E_x, E_y, 0)$ is an electrical field and $\vec{B}=(B_x, B_y, 0)$ is a magnetic field of the opposite bunch. Taking into account, that particles are relativistic $\beta \approx 1$, $\gamma >> 1$ and electromagnetic field in the interaction point is created by an opposite beam with round Gaussian space charge distribution, vector $\vec{P}$ is simplified:

$$P_x = 4G\frac{I}{I_c}\frac{y}{r^2}\,[1 - \exp(-\frac{r^2}{2\sigma^2})], \qquad (11)$$

$$P_y = -4G\frac{I}{I_c}\frac{x}{r^2}\,[1 - \exp(-\frac{r^2}{2\sigma^2})], \qquad (12)$$

where I is a beam current and $I_c = 4\pi\varepsilon_0\, m_0\, c^3/q = (A/Z)\cdot 3.13\cdot 10^7$ Amp is a characteristic value of the beam current.

3. Analytical Treatment of Spin Depolarization

To make an analytical treatment of spin depolarization, let us consider a collider with two installed Siberian Snakes and one interaction point. Matrix of spin advance after one revolution in the ring between beam-beam interaction is

$$M_{ring} = D_{\pi/2}\cdot S_2\cdot D_\pi\cdot S_1\cdot D_{\pi/2} = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix}. \qquad (13)$$

Suppose, the betatron tunes in x and y directions are equal each other $\bar{\theta}_x = \bar{\theta}_y = \bar{\theta}$. We will consider particle motion far enough from low order resonances, therefore, particle trajectory can be expressed as a linear oscillator with perturbed betatron tune θ:

$$x = r\cos(n\theta + \Psi), \quad y = r\sin(n\theta + \Psi), \quad \theta = \bar{\theta} + \Delta\theta, \qquad (14)$$

where Ψ is an initial phase of betatron particle oscillations and $\Delta\theta << 2\pi$ is tune perturbation due to beam-beam collisions. Parameters A and B in eq. (6) can be expressed as follow:

$$A = \frac{P_x}{P_o} = \sin(n\theta + \Psi), \qquad B = \frac{P_y}{P_o} = -\cos(n\theta + \Psi). \qquad (15)$$

Then, let us take into account, that parameter $\varphi = P_o\cdot\delta z$ in eq. (8) is small:

$$\varphi = P_o\delta z = \frac{2I\,l}{I_c\, r}G[1 - \exp(-\frac{r^2}{2\sigma^2})] \approx \frac{I\,l\,rG}{I_c\,\sigma^2}(1 + ..) << 2\pi. \qquad (16)$$

Hence, the matrix parameters a and b in eq. (8) can be approximated as follow:

$$a = 1 - \cos\varphi \approx \frac{\varphi^2}{2}, \qquad b = \sin\varphi \approx \varphi. \qquad (17)$$

Finally, matrix of spin advance of a particle in interaction point at the n-th turn is:

$$\begin{vmatrix} [1-\frac{\varphi^2}{2}\cos^2(n\theta+\Psi)] & [-\frac{\varphi^2}{4}\sin 2(n\theta+\Psi)] & [-\varphi\cos(n\theta+\Psi)] \\ [-\frac{\varphi^2}{4}\sin 2(n\theta+\Psi)] & [1-\frac{\varphi^2}{2}\sin^2(n\theta+\Psi)] & [-\varphi\sin(n\theta+\Psi)] \\ \varphi\cos(n\theta+\Psi) & \varphi\sin(n\theta+\Psi) & 1-\frac{\varphi^2}{2} \end{vmatrix}. \qquad (18)$$

It is easy to verify, that determinant of the matrix (18) equals unit, as it is required by conservation of absolute value of spin $|\vec{S}| = 1$.

Subsequent multiplication of matrixes (13) and (18) gives us possibility to predict effect of beam-beam interaction on spin depolarization after large number of turns. Suppose, initial spin vector has only one transverse component $S_y = 1$ and other components are equal to zero $S_x = S_z = 0$. Spin advance after n turns is as follow

$$S_x = \frac{\varphi^2}{4}[\sum_{i=0}^{n-1}(-1)^{i+n-1}\sin 2(i\theta+\Psi) + ...], \qquad (19)$$

$$S_y = 1 - \frac{\varphi^2}{2}[\sum_{i=0}^{n-1}(-1)^i\sin(i\theta+\Psi)]^2, \qquad (20)$$

$$S_z = (-1)^n\varphi\sum_{i=0}^{n-1}(-1)^i\sin(i\theta+\Psi), \qquad (21)$$

Average values of spin components are achieved by integration of eqs. (19-21) over all initial phases and averaging over turn number:

$$\bar{S}_x = 0, \quad \bar{S}_z = 0, \quad \bar{S}_y = 1 - \frac{1}{8}\left(\frac{\tilde{\varphi}}{\cos(\tilde{\theta}/2)}\right)^2, \qquad (22)$$

where $\tilde{\varphi}$ and $\tilde{\theta}$ are average values of parameters φ, θ among all particles:

$$\tilde{\varphi} = 4\pi G\gamma\xi\frac{\sigma}{\beta^*}, \qquad \tilde{\theta} = 2\pi(Q - \frac{\xi}{2}). \qquad (23)$$

Averaged root-mean-square values of spin components are given by

$$\overline{<S_x^2>} = [\frac{\tilde{\varphi}^2}{8(\cos\tilde{\theta})}]^2, \quad \overline{<S_y^2>} = \frac{3}{512}[\frac{\tilde{\varphi}}{\cos(\tilde{\theta}/2)}]^4, \qquad (24)$$

$$\overline{<S_z^2>} = [\frac{\tilde{\varphi}}{2\cos(\tilde{\theta}/2)}]^2. \qquad (25)$$

Attained formulas indicate, that spin depolarization due to beam-beam collisions is suppressed and depends on betatron tune in the ring. The most dangerous working point is close to half-integer value of betatron tune, because in that case the value of cos $(\tilde{\theta}/2)$ is close to zero and spin depolarization becomes large. Due to small value of φ, all effects, proportional to φ^4 are negligible as compare with that, proportional to φ^2. Therefore, among possible depolarization effects, the most pronounced are change of average value of S_y component and rms value of $<S_z^2>$.

4. Numerical Simulation of Beam-Beam Effect on Spin Depolarization

Computer simulations utilizing numerical model of Section 2 were performed for the beam parameters, presented in Table 1.

Table 1. Parameters of the interacted beams

Particle energy, γ	260
Rms beam size at interaction point (IP), σ	0.08 mm
Beam-beam tune shift per collision ξ	- 0.0125
Beta function of a ring, β^*,	0.65 m
Number of modeling particles, N	5000

For that combination of collider parameters, the value of matrix parameters are as follow:

$$\tilde{\varphi} = 4\pi G\,\gamma\,\xi \frac{\sigma}{\beta^*} = 9\cdot10^{-3}, \quad \tilde{\theta} = 2\pi\,(Q - 0.006)\,. \qquad (26)$$

Results of numerical simulations indicate that in presence of Siberian Snakes spin depolarization is suppressed. Depolarization is expressed as a manifestation of spin components distribution (see Fig. 1). Distribution of S_x component is much narrow than that of S_z component. It also follows from eqs. (24), (25), where $<S_x^2>$ is proportional to φ^4, while $<S_z^2>$ is proportional to φ^2. Average value of S_y oscillates around stable point close to unit (see Fig. 2). Average values of S_x and S_z oscillate around zero (not presented in Fig). The most significant depolarization is observed if betatron tune is close to half integer value. In other cases depolarization is substantially smaller. Numerical results are in good agreement with analytical predictions of the preceding section.

Without Siberian Snakes beam-beam interaction results in slow spin depolarization. In Fig. 3 average value of spin component S_y exhibit monotonous decreasing. It indicates, that presence of Snakes is important to prevent depolarization due to beam-beam collision.

References

[1]. A.Chao, NIM 180 (1981) p. 29.
[2]. T.Katayama, "Basic theory of spin dynamics in RHIC", RIKEN Internal Note.

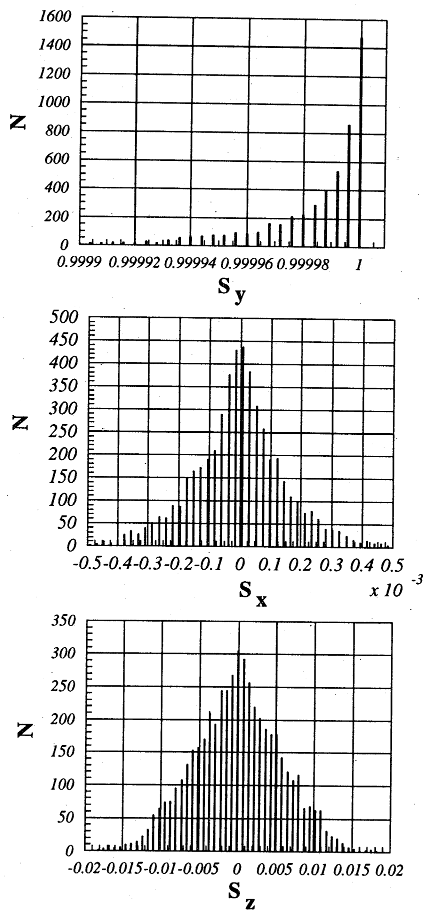

Fig. 1. Spin distribution after 500 000 turns in a ring with one IP and two Siberian Snakes, $Q_x = Q_y = 14.249$.

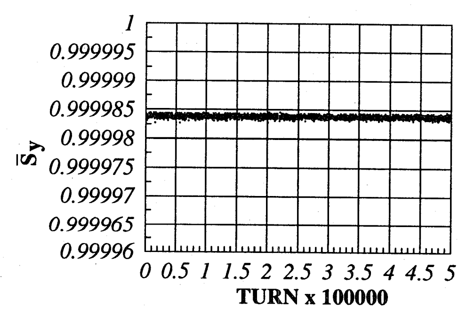

Fig. 2. Average value of spin component $\bar{S}_y$ as a function of turn number in a ring with one IP and two Siberian Snakes, $Q_x = Q_y = 14.249$.

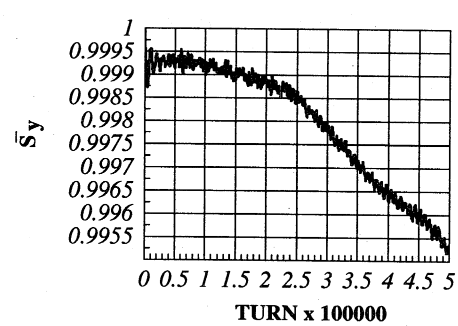

Fig. 3. Spin depolarization due to beam-beam collisions in a ring without Siberian Snakes, $Q_x = 14.095$, $Q_y = 14.59$.