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A Numerical Analysis of Noninertial Space-Charge Effect in X-ray Free-Electron Lasers

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Abstract

We have studied on the degradation of short electron bunches traveling through a circular orbit, which is a part of free-electron lasers (FEL). Transverse emittance growth is computed by using a particle-tracking simulation code based on a line-by-point space-charge force model. In this paper, we confirm that the line-by-point calculation implemented in the simulation code gives more accurate values for space-charge induced forces than the conventional point-to-point calculation.

1 Introduction

At present, FEL is considered as the only practical tunable coherent light source in the wavelength region shorter than 100 nm. While synchrotron radiation is available in the region, FEL will provide the coherent light in X-ray wavelength region with brightness several orders of magnitudes higher than that from synchrotron radiation. High-gain single-pass FEL is the only candidate to realize FEL operations in arbitrarily short wavelengths, because high-reflectivity normal-incidence mirrors do not exist. A configuration of self-amplified spontaneous emission (SASE) scheme [1], which can realize X-ray FEL, is shown in Fig.1. The basic concept is that the single pass gain is so high that the noise signal from spontaneous emission is amplified to intense coherent radiation by traveling with electron beam of high brightness in a long undulator. High-brightness electron beam is, therefore, required to obtain saturated laser power in single-pass FEL starting from noise signal. The essential point in the development of SASE X-ray FEL is generation of high-brightness electron: a laser driven RF photocathode gun, an emittance compensation technique and a beam transport without the degradation of beam quality. Recent studies have shown that two novel space-charge forces are induced in beams traveling through a circular motion, even at high energy [2]. One is called as noninertial space-charge force, where the energy of each particle is modified without total energy loss. The other is coherent synchrotron radiation force, where the bunch emits the radiation power coherently. Thus these space-charge forces may induce transverse emittance growth in high-quality beams passing through a bunch compressor and a long wiggler in X-ray FEL.

In our study, we make an analysis of the degradation of beam quality in a circular motion to confirm design requirement for X-ray FEL. The degradation of the beam performance is evaluated in section 2, where both the noninertial space-charge force and the radiation force are estimated by using the retarded Green's function for the scalar and vector potentials [3]. The validity of a line-by-point calculation for space-charge induced forces is studied in section 3, where the line-by-point calculation and a point-to-point calculation are compared by evaluating emittance growth of an electron bunch traveling a drift-space straight-line.

2 The total space-charge force from a line of charge

When a short line of charge is traveling in a circular motion, the electric field along the direction of motion is given by

$$E_\theta = -\frac{\partial A_\theta}{\partial t} - \frac{1}{r}\frac{\partial \phi}{\partial \theta}, \qquad (1)$$

where cylindrical coordinates is used and θ corresponds to the direction of motion.

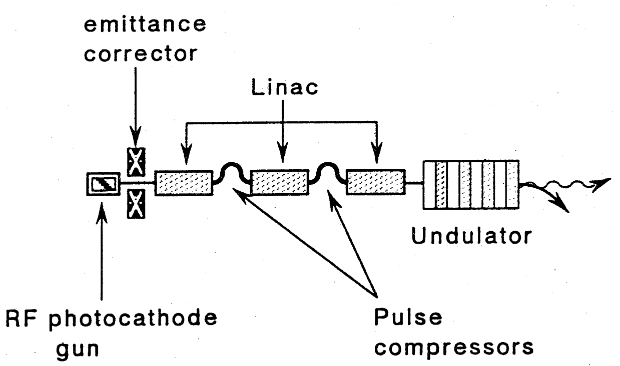

Fig.1 The general scheme of a linac based SASE generation

To evaluate E_θ induced from a line of uniform charge of length δ, we start with the scalar potential and the azimuthal component of the vector potential using the retarded potential:

$$\phi = -\frac{R}{4\pi\epsilon}\int_{\zeta_r}^{\zeta_f}\frac{\lambda}{r_{\rm ret} - \boldsymbol{r}_{\rm ret}\cdot\boldsymbol{u}_{\rm ret}/c}\,d\zeta \qquad (2)$$

and

$$A_\theta = -\frac{R}{4\pi\epsilon c}\int_{\zeta_r}^{\zeta_f}\frac{\lambda\,\beta\cos\zeta'}{r_{\rm ret} - \boldsymbol{r}_{\rm ret}\cdot\boldsymbol{u}_{\rm ret}/c}\,d\zeta, \qquad (3)$$

where ζ is the azimuthal angle relative to the observer location, $\lambda = Q/\delta$ is the current density, and ζ' is the retarded angle of the point's velocity, all shown in Fig.2.

Substituting of Eq.(2) and Eq.(3) into Eq.(1), we find the azimuthal electric field (to lowest order in x/R):

$$E_\theta = \frac{\lambda}{4\pi\epsilon}\,\frac{1}{r_{\rm ret} - \boldsymbol{r}_{\rm ret}\cdot\boldsymbol{u}_{\rm ret}/c} \times\left(\frac{1}{\gamma^2} - \beta^2\frac{x}{R} + \beta^2\left[1-\cos(\zeta')\right]\right)\Bigg|_{\zeta_r}^{\zeta_f}. \qquad (4)$$

Each of the three terms in parentheses is easily identified with a physical mechanism: the first term is the usual space-charge term, and vanishes if the beam is ultrarelativistic; the second term is the noninertial space-charge term; and the third term is the coherent synchrotron radiation term.

In order to evaluate the effect of the noninertial space-charge force and the coherent synchrotron radiation force, we need to find the retarded angle.

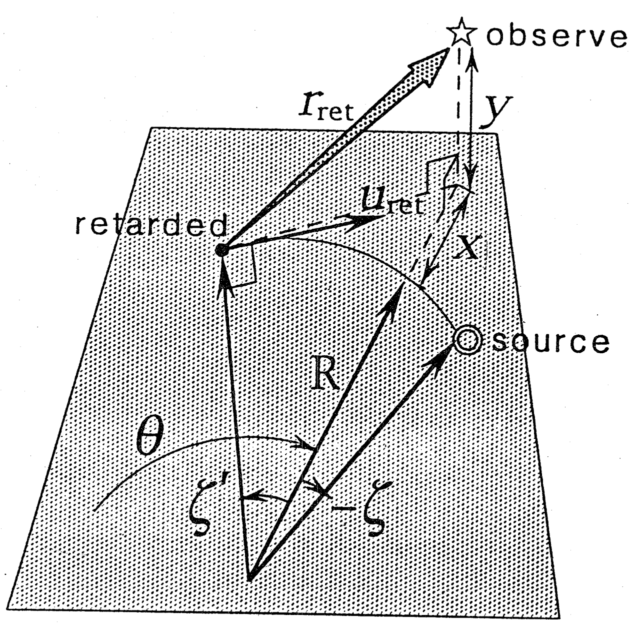

Fig.2 Geometry defining the observer position, the relative present-angle ζ, and the relative retarded-angle ζ'.

The retarded angle ζ' satisfies the transcendental equation,

$$\beta^2R^2(\zeta'-\zeta)^2 = \rho^2 + 2R(R+x)(1-\cos\zeta'). \qquad (5)$$

In Fig.3 we illustrate calculated retarded angle ζ' as a function of present angle ζ, with $x = 1$ mm, $y = 0$ mm, a radius of curvature of 1 m, and a beam energy of 100 MeV.

Figure 4 shows the coherent synchrotron radiation force and the noninertial space-charge force from one edge of a line charge as a function of the present angle ζ. It is confirmed that both of space-charge induced forces enlarge as the present angle is close to zero, and that the coherent synchrotron radiation force acts uniformly for the positive present-angle, where a source-point is located in front of the observer-point.

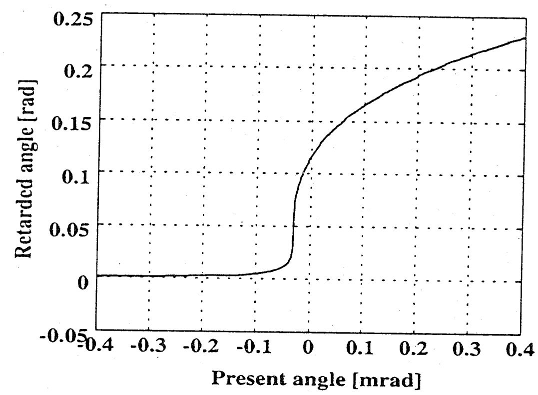

Fig.3 Relation between the retarded angle and the present angle according to the transcendental equation

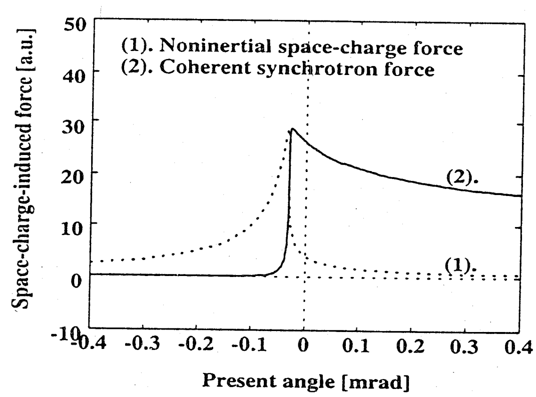

Fig.4 Two space-charge-induced forces as a function of the present angle ζ

3 Numerical simulations of a bunched-beam drifting in straight sections

Three dimensional space-charge induced force is usually calculated with the point-to-point method in analyses of particle motion in accelerators. Each particle in a bunched-beam receives space-charge force from the other particles,

$$F = \frac{e\,Q\,(\Delta x, \Delta y, \Delta z)}{n_{parts}\,4\,\pi\,\epsilon_0\,(\Delta x^2 + \Delta y^2 + \Delta z^2 + b^2)^{3/2}}. \quad (6)$$

where Q is the bunch charge, there are n_{parts} particles followed in the simulation, and b is impact parameter. Since the point-to-point calculation for space-charge forces causes some noise on particles' motion, we apply line-by-point method to this calculation, where the source particle is represented by an axially orientated line of charge. Addition of the particles' lengths is filled up with the bunch more uniformly and leads to less noise than representing the particle by a point of charge (Fig.5)

We compare the point-to-point calculation and the line-by-point calculation for space-charge induced forces by using a particle-tracking code we have developed, where a bunched-beam travels in a straight-line. In Fig.6 and Fig.7 the scaled transverse emittance growth of a bunched-beam is plotted as a function of drift length, where the geometry of a electron bunch is chosen as σ_z = 10.0 cm, r = 1.0 cm.

By changing the beam energy, the emittance growth of a drifting bunched-beam with the point-to-point method is scattering from the theoretical emittance growth [4]. On the other hand, the emittance growth with the line-by-point method shows good agreement with the theoretical emittance growth independent of beam energy and current.

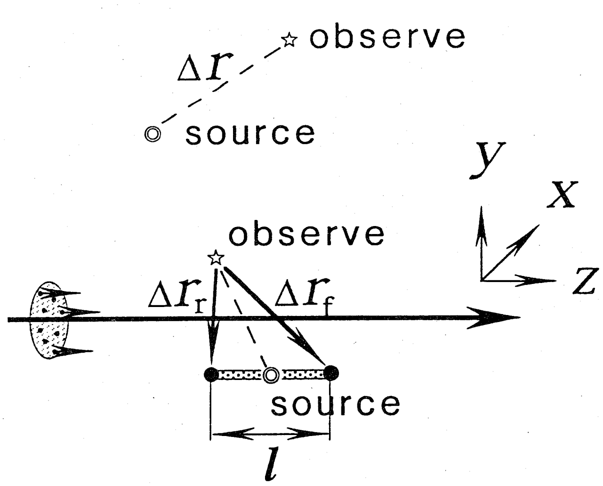

Fig.5 Geometry defining a point of charge and a line of charge

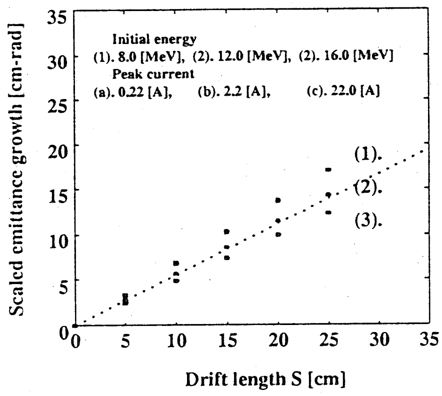

Fig.6 Emittance growth of a drifting bunched-beam in the point-to-point calculation

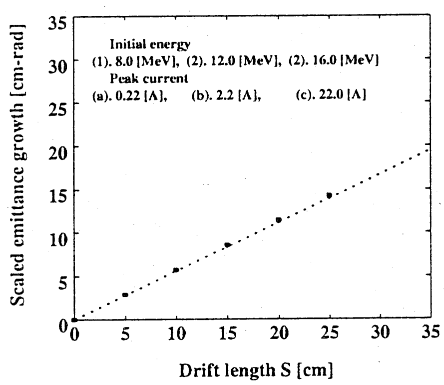

Fig.7 Emittance growth of a drifting bunched-beam in the line-by-point calculation

4 Conclusion

We have estimated both the noninertial space-charge effect and the radiation effect in a bunched-beam traveling a circular motion. We also confirm that the line-by-point calculation for space-charge induced forces gives more accurate values for numerical simulations of a drifting bunched-beam than conventional point-to-point method. In future, we will apply the particle-tracking code to the numerical simulation of a bunched-beam traveling in a circular orbit, and estimate the transverse emittance growth of high-quality beam in X-ray FEL.

References

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