

# Study of Stochastic Cooling in the Accumulator Cooler Ring at MUSES Project

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## Abstract

We studied availability of a stochastic cooling for a cooling accumulation of RI beam in the ACR. In the ACR fast cooling is essential for an RI beam with large emittance and momentum spread because the RI has an intrinsic life time. In general the stochastic cooling is useful to obtain a fast cooling time in a region where emittance and momentum spread are large. From calculations under some conditions, we could get a very short cooling time ( $< 0.1$  s) and confirm that the stochastic cooling is available with cooling of the RI beam in the ACR.

## 1. Introduction

One of purposes for the accumulator cooler ring [1] (ACR) at the MUSES project[2] is cooling of an ion beam including a radio-isotope (RI) beam produced by projectile-fragmentation in the Big-RIPS. Due to production mechanism the RI beam has large emittance ( $\epsilon_x$ ; horizontal,  $\epsilon_y$ ; vertical) and momentum spread ( $\Delta p/p$ ). Moreover, due to multi-turn injection,  $\epsilon_x$  of the beam injected into the ACR is very large. In the ACR initial values of  $\epsilon_x$ ,  $\epsilon_y$  and  $\Delta p/p$  are  $125 \pi$  mm\*mrad,  $10 \pi$  mm\*mrad and  $\pm 0.15$  %. Requirement of cooling in the ACR is to have a very fast cooling time for the RI beam with large  $\epsilon_x$ ,  $\epsilon_y$  and  $\Delta p/p$  because the RI has an intrinsic life time.

Stochastic cooling[3] is one of useful methods for cooling of the ion beam. In particular, a fast cooling time is obtained by the stochastic cooling for in a region of large  $\epsilon_x$ ,  $\epsilon_y$  and  $\Delta p/p$  as the initial beam in the ACR. On the other hand, in a region of small  $\epsilon_x$ ,  $\epsilon_y$  and  $\Delta p/p$ , the cooling time of the stochastic cooling is longer than that of electron cooling. In this way the stochastic cooling will be used as pre-cooling that reduces initial large  $\epsilon_x$ ,  $\epsilon_y$  and  $\Delta p/p$  to about 0.1 times. After pre-cooling the electron cooling will be used to make final small values of  $\epsilon_x$ ,  $\epsilon_y$  and  $\Delta p/p$ .

We calculated transverse and longitudinal cooling times of the stochastic cooling in several

conditions. In this paper we will mention calculations and results. Availability of stochastic cooling will be also discussed.

## 2. Transversal cooling

The transversal cooling time of the stochastic cooling is given by[4],

$$\frac{1}{\tau} = -\frac{W}{N} \left\{ -g_1 \sin \Delta \mu + \frac{1}{2} g_1^2 F \right\} \quad (1)$$

where

- W ; band-width of a feed back system
- N ; total number of particles
- $\Delta \mu$  ; phase advance between pickup and kicker
- $g_1$  ;  $g \sqrt{\beta_p \beta_k}$ ,  $g$  is gain of a feed back system and  $\beta_p$  and  $\beta_k$  is  $\beta$  values of pickup and kicker
- F ;  $M+U$ ,  $M$  is mixing factor and  $U$  is noise factor given, (noise of system) divided (beam noise per Schottky band)

Using parameters of a beam and a system,  $g$  is given

$$g = \frac{(qe)^2 f_0 Z_p f_k N G_{amp}}{\beta E W} \quad (2)$$

where

- q ; charge of beam
- $f_0$  ; revolution frequency
- $Z_p$  ; coupling impedance between beam and pickup
- $f_k$  ; efficiency of kicker
- $G_{amp}$  ; gain of amplifier
- $\beta$  ; Lorentz parameter of beam
- E ; energy of beam

From (1) and (2), we can get an optimum gain ( $G_{amp,opt}$ ) to get a minimum cooling time as shown in (3)

$$G_{amp,opt} = \frac{\sin \Delta \mu}{(M+U)} \frac{W}{N} \frac{\beta E}{(qe)^2 f_0 Z_p f_k} \quad (3)$$

Output power of an amplifier for  $G_{amp\_opt}$  is given

$$P = k(T + T_n) \left(1 + \frac{1}{U}\right) W G_{amp\_opt}^2 \quad (4)$$

where,  $k$  is Boatsmann constant and  $T$  and  $T_n$  are temperatures of the amplifier and a pickup system including a pre-amplifier. Using eq. (4) we calculated  $P$ 's for  $^{11}\text{Be}^{4+}$  (400 MeV/u),  $^{39}\text{Ca}^{20+}$  (300 MeV/u),  $^{132}\text{Sn}^{50+}$  (200 MeV/u) and  $^{200}\text{Pb}^{80+}$  (100 MeV/u) with  $\Delta p/p = \pm 0.15\%$  and  $\epsilon_x = 100 \pi$  mm\*mrad. In the calculation we assumed  $T_n = 20$  K,  $W = 2$  GHz and  $Z_p = 100 \Omega$ . The other parameters of the ACR are shown in ref [1]. Results are shown in Fig. 1 in term of dependence of number of particles. As shown in Fig. 1, the obtained  $P$ 's are very large especially for small number of particles as RI beam. From the result, we studied the cooling times under limited power of the amplifier ( $P_{lim}$ ).

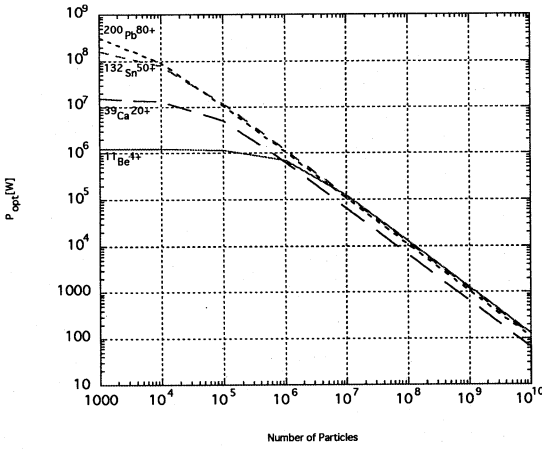


Fig. 1 Output power of an amplifier.

In a power-limited system the cooling time is given

$$\frac{1}{\tau_{lim}} = \frac{1}{\tau_{opt}} \frac{G_{amp\_lim}}{G_{amp\_opt}} \left(2 - \frac{G_{amp\_lim}}{G_{amp\_opt}}\right) \quad (5)$$

where, subscript "lim" means the limited system and  $\tau_{opt}$  is a minimum cooling time obtained from eqs. (1), (2) and (3). In eq. (5)  $G_{opt\_lim}$  has a value obtained under the condition  $P = P_{lim}$  in eq. (4).

Under the assumption of  $P_{lim} = 10$  kW we calculated the cooling times for several beams with several number of particles. In Fig. 2 the results are shown. As shown in Fig. 2, the obtained cooling time is less than 0.1 s for particle number of  $10^3 \sim 10^6$  as RI beam. In particular for beam with a high charge the cooling time is very fast.

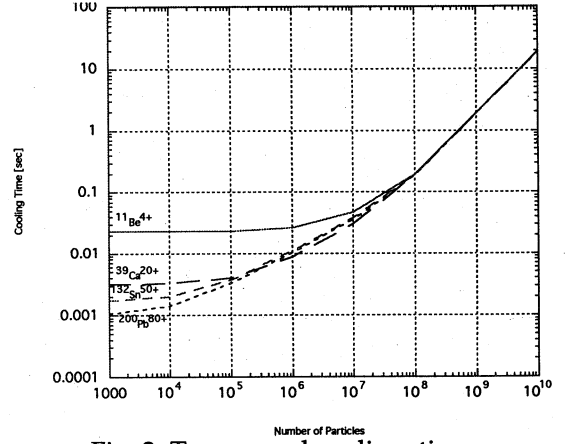


Fig. 2 Transversal cooling time.

### 3. Longitudinal cooling

We also studied longitudinal cooling time by a notch filter method[5] which cooling is done in a region of dispersion free. Similar to the transversal cooling, the cooling time of the longitudinal cooling is limited by output power of the amplifier. In the power-limited system the cooling time is given by the same equation as eq. (5). For the longitudinal cooling  $\tau_{opt}$  is given

$$\tau_{opt} = \left(\frac{N}{W}\right) \left(1 + \frac{W^2}{12W_c^2}\right) (1 + U(0)) \quad (6)$$

where  $W_c$  is average frequency of the amplifier and  $U(0)$  is give by

$$U(0) = \left(1 + \frac{W^2}{12W_c^2}\right)^{-1} \left(\frac{f_0}{\kappa W_c} \frac{E_0}{\Delta E_0}\right)^2 U_p \quad (7)$$

where,  $\Delta E$  is spread of energy and

$$\kappa = \frac{1}{\gamma} - \frac{1}{\gamma_t} \left(\frac{\gamma}{1 + \gamma}\right)$$

where  $\gamma$  is Lorentz parameter and  $\gamma_t$  is transition  $\gamma$ .  $G_{amp\_opt}$  and  $P$  for the longitudinal cooling are given by

$$G_{amp\_opt} = \frac{1}{2} \frac{A}{(qe)^2} \frac{1}{NW_c} \left(1 + \frac{W}{12W_c}\right)^{-1} \frac{E_0}{|\kappa Z_p f_k|} \frac{1}{1 + U(0)} \quad (8)$$

$$P = \frac{1}{3} k(T + T_n) W G_{amp\_opt}^2 \left(1 + \frac{1}{U(0)}\right) \quad (9)$$

where  $A$  is a mass number of the beam. The other parameters which don't explain in eqs. (6) ~ (9) are the same as the transverse cooling.

We calculated particle number dependence of the cooling time. Kinds of particles and conditions are the same as those used in the transversal cooling. In Fig. 3 results are shown. For number of particles  $< 10^7$  the obtained cooling times are less than 0.1 s. In particular the cooling time of particle with high charge is very fast. These tendencies are similar to the transversal cooling.

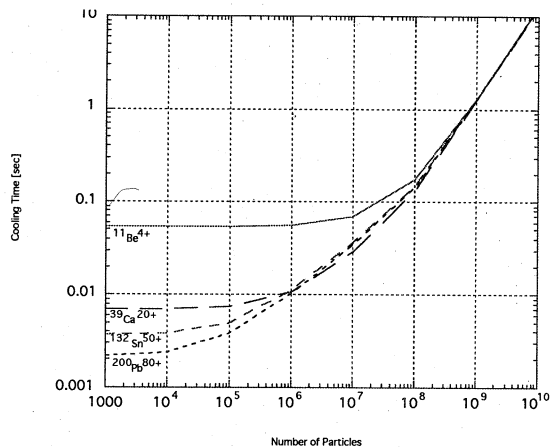


Fig. 3 Longitudinal cooling time.

We also calculated cooling times for several values of temperatures of a pickup system ( $T_n$ ) and coupling impedances ( $Z_p$ ). Fig. 4 and 5 show  $T_n$  and  $Z_p$  dependences. In both calculations we assumed number of particles  $10^5$ . For calculation of  $T_n$  dependence we assumed  $Z_p = 100 \Omega$  and for that of  $Z_p$  dependence,  $T_n = 20 \text{ K}$ . As shown in Fig. 4 the  $T_n$  dependence is weak. Cooling times of highly charged beams are shorter than 0.01 s even in 100 K. It is due to good signal-to-noise ratio because voltage induced by the highly charged beam at the pickup is much larger than that by noise. As shown in Fig. 5  $Z_p$  dependence is strong especially for region below  $50 \Omega$ . To obtain  $\tau < 0.1 \text{ s}$   $Z_p > 50 \Omega$  is required.

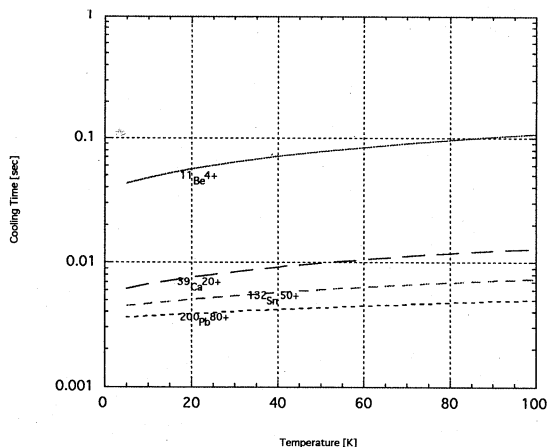


Fig. 4  $T_n$  dependence of cooling time. Here, number of particles is  $10^5$ .

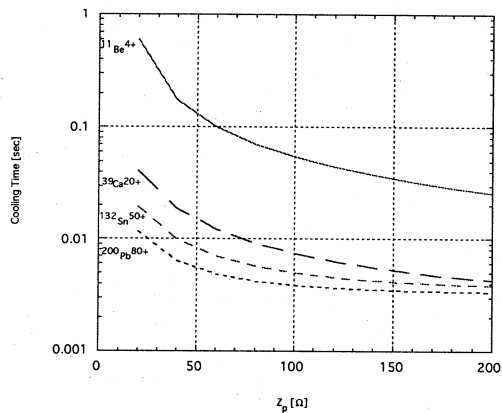


Fig. 5  $Z_p$  dependence of cooling time. Here, number of particles is  $10^5$ .

#### 4. Availability of stochastic cooling

From calculations in sec. 2 and 3 we obtained fast cooling times in almost whole region. This indicates that the stochastic cooling is available with the cooling of the RI beam in the ACR.

One of the important things to construct a system of the stochastic cooling in the ACR is to make a pickup system with large coupling impedance. The large coupling impedance is also important to reduce a needed output gain of the amplifier because a voltage from pickup itself is large. On the other hand it is not necessary for the pickup system to be very low temperature. This makes construction of the system to be easy.

On the base of the calculation we are designing the pickup system with high coupling impedance.

#### References

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