A Possible 3D Laser Cooling Scheme in a Storage Ring

Takahiro KIHARA, Hiromi OKAMOTO and Yoshihisa IWASHITA
Accelerator Laboratory, Institute for Chemical Research, Kyoto University,
Gokanoshou, Uji, Kyoto, 611 Japan

Abstract

We describe a practical method to realize the three-dimensional laser cooling of fast circulating beams in a storage ring. Synchrobetatron coupling is used to extend the longitudinal laser cooling force to the transverse degrees of freedom. In particular, we discuss the advantage of introducing a coupling cavity, showing the possible geometry. The preliminary results of molecular dynamics simulations are presented to demonstrate the transverse equilibrium temperatures achievable with the proposed scheme.

1 Introduction

Laser cooling has been well-known as a powerful tool to attain ion beams with extremely high phase-space densities[1][2]. It has already been experimentally demonstrated that one can produce an ultra-cold beam close to the longitudinal space-charge limit[3][4]. Direct transverse cooling is, however, not available for fast circulating beams due to the poor interaction between the transverse laser and a beam. The equilibrium temperatures of the horizontal and vertical direction are thus generally much higher than that of the longitudinal direction.

To overcome this difficulty of the laser cooling technique, a novel method has been proposed in the past papers[5][6]. The idea is simple, that is, we take advantage of a synchrobetatron resonance to increase the transverse cooling rates in an indirect way. So far, two separate schemes, i.e. the coupling-cavity scheme and the dispersion-coupling scheme, have been theoretically and numerically explored in order to incorporate the idea in a real storage ring. According to the single-particle theory, the horizontal cooling rate can be most enhanced under the condition

$$\nu_x - \nu_s = \text{integer}, \tag{1}$$

where ν_x and ν_s are, respectively, the horizontal and longitudinal tune. Needless to say, the similar condition must be satisfied between ν_x and ν_y to get a sufficiently large vertical cooling rate.

Specifically, the dispersion-coupling scheme is, in a practical sense, much simpler than the coupling-cavity scheme as we only need to put a regular rf cavity in a region where there is momentum dispersion. Nevertheless, the coupling-cavity scheme is of importance since it is less sensitive to the resonance condition in Eq.(1). Further, a vertical coupling cavity enables us to provide a direct coupling between the longitudinal degree of freedom and the vertical one.

In this paper, we consider the lattice of the ASTRID ring in Denmark where a laser-cooling device has actually been installed. After giving a possible design of a coupling cavity, we approximately incorporate the corresponding rf fields into the molecular dynamics (MD) code in order to evaluate the equilibrium beam temperature achievable with the scheme.

2 A Possible Coupling-cavity Design

The ideal vector potential theoretically required for the coupling cavity is represented as

$$\boldsymbol{A} = (0, 0, V_0 x \sin \omega t), \tag{2}$$

where V_0 is constant, x is the horizontal coordinate measured from the closed orbit, and ω is the operating frequency of the cavity. For a verical-to-longitudinal coupling, the cavity is simply rotated by 90 degrees around the longitudinal axis, replacing x in Eq.(2) by the vertical coordinate y. (Notice here that, in Eq.(2), we have used the paraxial approximation, so this potential does not satisfy the Maxwell equations.)

It is straightforward to show that a simple rectangular resonator operating in the TM210 mode can be employed to obtain the rf potential in Eq.(2). However, this is generally impracticable because of the rather low operating frequency desirable for the coupling cavity. In fact, a frequency in the range less than 100 MHz would be preferred in many applications. We are then forced to employ a lumped circuit as illustrated in Fig.1. The axial length of this type of the coupling cavity is roughly equal to $2\beta\lambda$ where λ is the rf wavelength and β is the velocity of the reference particle relative to the speed of light. As indicated in the figure, there exists a dipole-like electric field in-between the two electrodes.

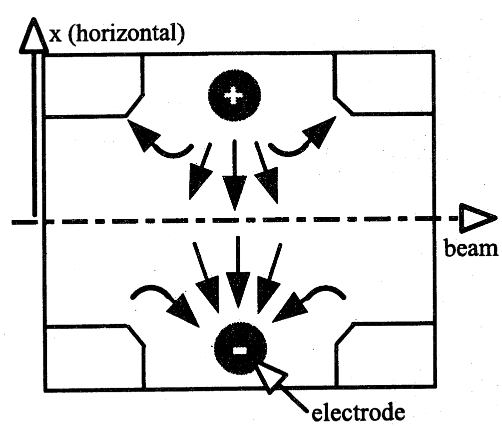

Fig. 1 A possible geometry of the coupling cavity. Voltages of opposite signs are applied to the two electrodes.

Since $\beta \ll 1$ in most laser-cooled ions, the cavity dimension is much smaller than the wavelength l. We are then allowed to use the code POISSON to calculate an approximate rf field of the coupling cavity, treating the problem as electrostatic[7]. The equipotential lines corresponding to the structure in Fig.1 are shown in Fig.2. The effective longitudinal electric field numerically evaluated from the POISSON output is given in Fig.3. It is now clear that the longitudinal component of the cavity potential has a nice linearity with respect to x.

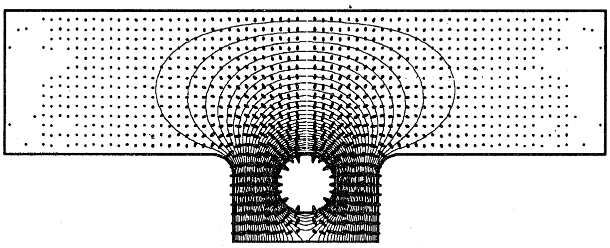

Fig. 2 The approximate electric field and equipotential line of the coupling cavity. Only lower half of Fig.1 had been shown.

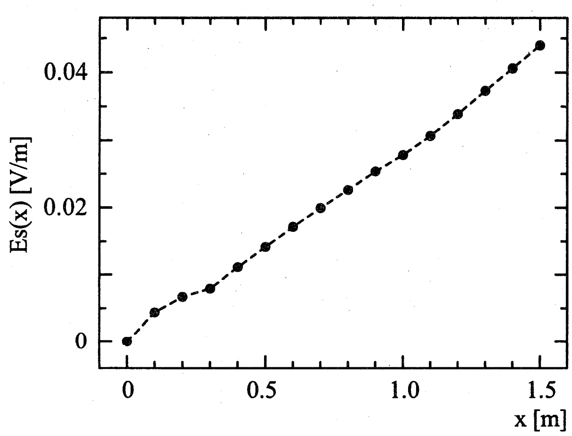

Fig. 3 The effective electric field strength of the coupling cavity vs. the transverce coordinate x. The absolute value of the electrode voltage has been set at 1 volt in this example.

3 Molecular Dynamics Results

As the beam temperature is reduced, by cooling, to a lower level, the phase-space density of the beam becomes higher. Consequently, the tunes are more and more depressed due to the space-charge repulsive force, leading to the breakdown of the resonance conditions. Since, according to the single-particle theory, the transverse cooling rates cannot be well enhanced without the resonance conditions approximately satisfied, it is expected that the coupling mechanism may cease working before we reach an ultra-cold state. It is thus important to figure out the limitation of the coupling scheme by employing the MD approach.

In this section, we show some preliminary results of MD simulations. The details of the MD calculations are available in Ref.[8]. As an example, we here take into account the exact lattice parameters of the ASTRID ring at Aarhus University in Denmark, since it is one of the two storage rings where extensive laser cooling experiments have been performed. (Another storage ring with a laser cooler is TSR in Heiderberg, Germany.) Further, we assume $^{24}Mg^{+}$ ions stored at the total kinetic energy of 100 keV, considering the recent experiments in ASTRID. For the sake of simplicity, we only try, in this paper, the 2D cooling scheme with and without the horizontal coupling cavity. In order to extend the present calculations to 3D, all we must do is simply either to turn on a skew quadrupole magnet or to introduce a vertical coupling cavity.

As is obvious from the condition in Eq.(1), we always need a regular rf cavity to establish an enhanced synchrobetatron coupling. Needless to say, the synchrotron tune ns should be sufficiently large to avoid the excitation of integer resonances in the transverse motions. But, on the other hand, we prefer as low an rf voltage applied to a laser-cooled beam as possible. To meet these requirements simultaneously, we here set the operating frequency of the cavity to be 5.72 MHz corresponding to the harmonic number of 256. This value is about ten times higher than that used in the usual ASTRID experiments. The rf frequency of the coupling cavity is taken to be the same as the regular-cavity frequency.

Fig.4 shows the equilibrium beam temperatures of horizontal and longitudinal degree of freedom reached after the longitudinal cooling process. The horizontal and

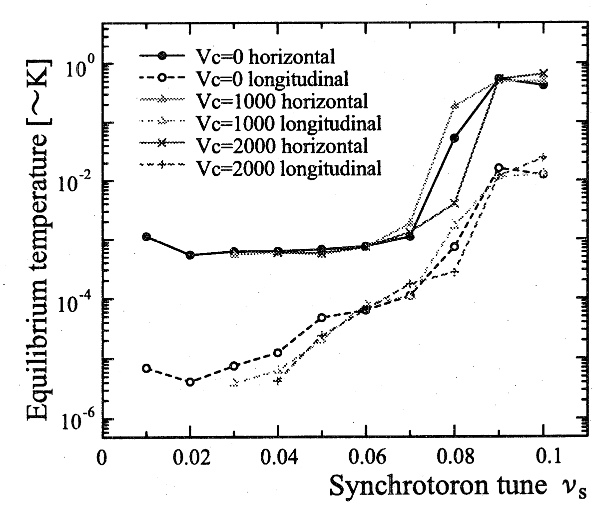

Fig. 4 Equilibrium beam temperature obtained with the MD method vs. synchrotron tune. Only ten magnesium ions stored in the ASTRID lattice have been considered in these MD simulations.

vertical tune have been set, respectively, at 3.1 and 2.73 in this example. Surprisingly, the achievable temperature has become lower in the region $\nu_s < 0.07$ though the operating point of the ring is on resonance at $\nu_s = 0.1$. As demonstrated in Fig.5, the horizontal cooling rate numerically evaluated from the MD simulations is actually maximum at $\nu_s = 0.1$ where the condition in Eq.(1) is exactly satisfied. The reachable temperature appears to be insensitive to whether the coupling cavity is switched on, while the coupling cavity can considerably enlarge the horizontal cooling rate.

Specifically, the horizontal equilibrium temperature in the off-resonance region shown in Fig.4 is roughly five orders of magnitude lower than that accomplished by the past experimental efforts, although we have only taken ten simulation particles in this example. Increasing the particle number to fifty, we have the result in Fig.6. The final temperature has been pushed up due to the stronger space-charge force, but it is still possible to reach the beam temperature which has never been achieved in any real storage rings.

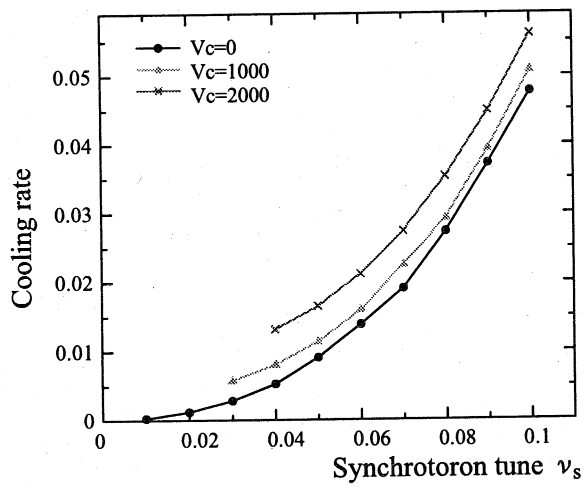

Fig. 5 Horizontal cooling rate evaluated from the MD simulation data vs. synchrotron tune.

4 Summary

We have confirmed, with the MD approach, that the transverse cooling rates can considerably be increased with the dynamical coupling provided either by a coupling cavity or by momentum dispersion at the position of a regular rf cavity. The cooling time is, of course, very much shorter than that of the sympathetic cooling relying upon intra-beam scatterings[9]. Further, according to the MD results, the transverse equilibrium temperature achievable with the coupling scheme is anticipated to be much lower than that having been experimentally attained so far, while only small numbers of the MD particles have been considered in this paper to reduce the computing time. More extensive and systematic MD simulations would be necessary to predict a more accurate value of reachable beam temperature.

As shown in Figs.4 and 6, we have found that the equilibrium temperature in a off-resonance region is even lower than that in the region close to the coupling resonance, as long as the present example is concerned. Although the transverse cooling rates become small without the resonance conditions, we can sufficiently enhance the transverse cooling rates by means of a coupling cavity. Note also that a vertical coupling cavity can even provide a direct coupling between the longitudinal and vertical degree of freedom.

To conclude, it seems that the coupling-cavity scheme offers us a great possibility to realize fast circulating ion beams in an ultra-cold state.

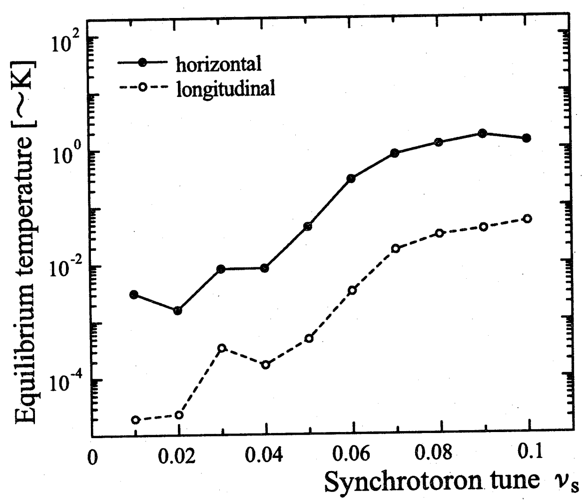

Fig. 6 Equilibrium beam temperature obtained with the MD method vs. synchrotron tune. Fifty simulation particles has been taken into account. The coupling cavity has been switched off.

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