BEAM-DYNAMICAL EFFECTS OF A DROOP IN AN INDUCTION ACCELERATING VOLTAGE

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Abstract

Proof-of-principle (POP) experiments on the induction synchrotron concept are scheduled using the KEK 12-GeV proton synchrotron, in which RF bunches and a super-bunch will be accelerated with a long step voltage generated in the induction accelerating gaps. An unavoidable droop in the induction voltage gives an additional focusing or defocusing force in the longitudinal direction. It largely deforms the barrier bucket confining the super-bunch, leading to a non-uniform particle distribution. The effects are serious in an induction synchrotron with a transition energy. Longitudinal emittance blow-up beyond the transition energy is not acceptable. The necessity of compensating for the droop is discussed.

INTRODUCTION

Super-bunch acceleration is a key concept in an induction synchrotron[1-2]. In an induction synchrotron, super-bunches confined in the longitudinal direction by a pair of barrier voltages are accelerated with long induction step voltage pulses.

Experiments for proof-of-principle (POP) of the induction synchrotron are planned using the KEK 12-GeV proton synchrotron during 2003-2007[3]. The experiments will proceed step by step. In the first step, a single RF bunch that is captured in the existing RF will be accelerated with the induction voltage alone. For this purpose, 4 newly developed induction accelerating cavities[4] with an output voltage of 2.5kV/each will be installed during the 2003 winter shutdown. As the second step, an induction barrier experiment is planned, where 1-9 booster RF bunches are injected into the main ring, immediately captured in the induction barrier bucket, and then merged into a single super-bunch. Various beam-handling exercises, such as adiabatic moving of the super-bunch, will be performed. In the last step, a super-bunch will be accelerated up to the flat-top energy with the induction voltage.

The induction acceleration device can be thought of as a series of one-to-one transformers driven by low-impedance pulse sources in which the beam acts as the secondary. In an equivalent-circuit model, the induction cavity is a parallel circuit of inductance, resistance, and capacitance, standing for the magnetic core, core-loss and other losses, and acceleration gap, respectively. The cavity is connected to a high-voltage pulse modulator through a matching cable, and driven at a repetition rate on the order of MHz[5]. Some droop in the output voltage, which is generated across the inductance, is generally unavoidable because of a finite inductance and resistance. Our preliminary measurement has indicated a droop of several to ten percent. It is easily supposed that the droop voltage gives accelerating particles an extra defocusing effect in the longitudinal direction below the transition energy and a focusing effect above the transition energy. It tends to deform the desired barrier bucket shape. The purpose of this paper is to theoretically manifest its beam dynamical effects and estimate its significance in the synchrotron oscillation of the super-bunch. In addition, a discussion will develop concerning the case of induction acceleration of RF bunches, to which we will soon face.

THEORY OF THE SYNCHROTRON MOTION PERTURBED BY A DROOP VOLTAGE

An induction accelerator is a pulsed passive device. In order to generate a step voltage in the accelerating gaps, the accelerating cavity is energized with a pulse modulator employing power MOS-FETs as switching elements, which are connected to a DC power supply. Switching of the modulator has to be synchronized with the revolution of beams; it is operated at a high repetition rate on the order of 1MHz.

The droop in the induction voltage is intrinsic when the step voltage is inputted and has an almost negatively linear gradient. As mentioned in Introduction, the droop voltage could cause an extra focusing and defocusing in the longitudinal motion of particles. In this section, the theory of synchrotron oscillation perturbed by a droop is developed, based on difference equations describing the longitudinal motion, which also serve for particle tracking. For the convenience of mathematical formulation, the induction units for acceleration are assumed to be placed near to the other units for particle confinement, which are RF cavities or induction units. This means that particles see two types of voltages excited in both devices at the same time in a ring.

For simplicity, the barrier voltage generated in the induction gaps, which is employed for the longitudinal confinement of the super-bunch, is assumed to be a step function in time,

$$V_c(\phi) = \begin{cases} -V_{step} & (\phi < -\phi_0) \\ 0 & (-\phi_0 \le \phi \le \phi_0), \\ V_{step} & (\phi > \phi_0) \end{cases}$$

where $\phi = \left\{ h \int \omega_s dt \right\} \mathrm{mod}(2\pi)$ represents the fractional phase from a synchronous phase ($\phi = 0$), $T_s = 2\pi/\omega_s$ is the revolution period of the synchronous particle, h is the harmonic number and V_{step} is the peak voltage. The sign of the voltage must be changed beyond the transition to maintain the phase stability, just as in a conventional RF synchrotron. If the droop of the accelerating voltage V_a is assumed to be linear in time as $dV_a/dt = k = -V_0Rt/(2L)$ [6], where L and R represent the core inductance and the total resistance of the core loss and the matching load, the accelerating voltage is written as

$$V_a(\phi) = V_0 + kt = V_0 + hk\int_0^{\phi} \frac{d\phi'}{\omega_s}.$$

For a synchronous particle, the temporal evolution of its energy from the nth turn to the $(n + 1)$th turn is given by the following difference equation:

$$(E_s)_{n+1} = (E_s)_n + eV_0,$$

where E_s is the energy of a synchronous particle placed at the middle point of the barrier voltage pulses. For an arbitrary particle, the corresponding equation is described by

$$E_{n+1} = E_n + e[V_c(\phi) + V_a(\phi)].$$

Thus, the energy difference from E_s, $\Delta E = E - E_s$, satisfies

$$\Delta E_{n+1} = \Delta E_n + e\left[V_c(\phi_n) + hk\int_0^{\phi_n} \frac{d\phi'}{(\omega_s)_n}\right]. \quad (1)$$

Meanwhile, the temporal-evolution of the fractional phase for the particle of interest is well-known to be given by

$$\phi_{n+1} = \phi_n + 2\pi\eta_{n+1}h\frac{\Delta E_{n+1}}{\beta_{n+1}^{\ 2}(E_s)_{n+1}}, \quad (2)$$

where η and β are the slippage factor and the relativistic beta of the design particle, respectively. A set of difference equations, Eq.(3) and (4), describes the synchrotron oscillation with the droop and becomes a theoretical basis of the following discussion.

If the parameters, such as η, β and ω_s, are assumed to be constant in n, the Hamiltonian is easily evaluated by Eq.(3) and (4) in the following form:

$$H(\phi, W) = \frac{W^2}{2} + U(\phi),$$

$$U(\phi) = -\frac{e\beta^2 E_s}{2\pi\eta\omega_s^{\ 2}h^3}\left[\int V_c(\phi)d\phi + \frac{hk}{2\omega_s}\phi^2\right], \quad (3)$$

where $W = \Delta E/(\omega_s h)$ and $U(\phi)$ represents the potential including effects caused by the droop. The effect originating from the droop is understood to be a simple quadratic potential there. As mentioned in Introduction, this term changes its role beyond the transition energy. It takes defocusing before the transition and focusing after the transition from a macroscopic view. In Figs.1, the contours of the Hamiltonian are shown for both cases. Particles tend to be trapped around $\phi = \pm\phi_0$ before the transition energy and a shallow hollow at its center after the transition energy, and a global modification of the super-bunch is predicted. Meanwhile, the potential for RF confinement looks like that of the moving RF bucket, because the position of the potential bottom changes with the phase. The contours of the Hamiltonian indicate that the stable points of the RF buckets exist at different locations from $2m\pi$, where m is an integer. This suggests that above synchronous phase compensates an extra energy gain and loss due to the droop.

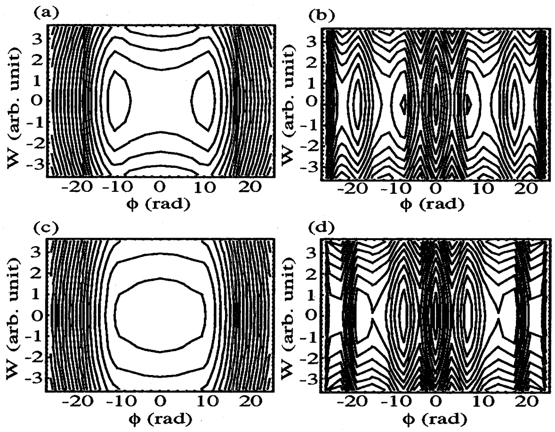

Figure 1: Contour of Hamiltonian for (a) the super-bunch beam and (b) the RF-bunch beam before the transition, and (c) the super-bunch and (d) the RF-bunch after the transition. $\phi_0 = 4\pi$ (rad), $k = -0.43$ (kV/μsec).

PARTICLE TRACKING

In order to delineate the longitudinal motion under acceleration by a long step voltage with a droop, particle tracking based on Eq.(1) and (2) has been performed. Most of the machine parameters were taken from that of the KEK-PS. The temporal evolution of V_0 and the kinetic energy are shown in Fig.2, which naturally follow the field pattern of the bending magnets. Two cases of (1) step-barrier confinement of 85kV and (2) conventional RF confinement of 92kV were carefully examined. A novel transition crossing method, in which the particles do not see the confinement voltage during a short time period before and after the transition energy (the detail is written in Ref.[6]), was employed.

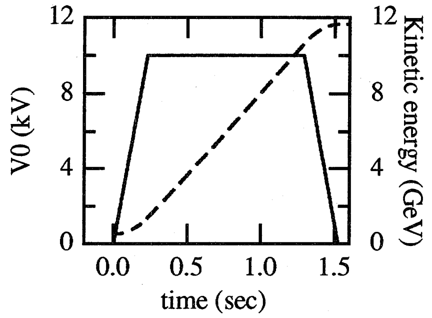

Figure 2: V_0 (solid) and the kinetic energy (dashed line).

For a super-bunch, the temporal evolution of the particle distribution on phase-space (ϕ, $\Delta p/p$) is shown in Figs.3. It is notable in Fig.3a that the super-bunch tends to split in phase-space before the transition energy. As predicted by Eq.(3), the potential depth becomes deeper as η becomes smaller. We understand that the split of the super-bunch is caused by trapping a substantial fraction of particles in double wells. On the other hand, the complicated structure in the projection on the momentum axis is notable after the transition (see. Fig.3b). This is explained by quadrupole oscillation due to the mismatching before and after the transition (see Fig.1a and 1c). This means that the droop effect is not time-reversible about the transition.

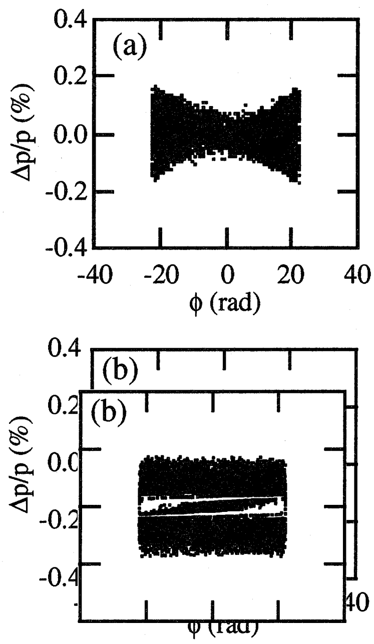

Figure 3: Phase-space projection of a super-bunch beam at (a) 4.69GeV and (b) 11.64GeV. $\phi_0 = 7\pi$ (rad) and k = -0.43 (kV/μsec).

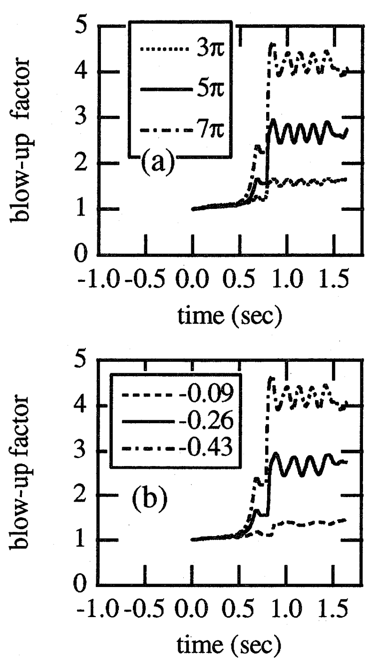

Figure 4: Blow-up factor of the emittance in the dependence on (a) ϕ_0 (rad) where k is set to -0.43 (kV/μsec), and (b) k (kV/μsec) where ϕ_0 is set to 7π(rad).

In a practical sense, there is a more significant aspect to which we must pay attention: emittance blow-up. The dependence of the emittance blow-up on the droop size and the length of the super-bunch have been examined. The results are shown in Figs.4. The emittance becomes larger as the droop size becomes larger, or the length becomes longer. This may be explained by the fact that the depth of the potential wells before a transition is simply proportional to the size of the droop.

In the case of the RF-bunches, no drastic phenomenon, such as in a super-bunch beam, is found. This is understandable from a fact that the droop size over the bunch length is relatively small compared with the RF defocusing/focusing voltage before/after a transition [6].

CONCLUSION

For a super-bunch beam and a RF-bunch beam, the longitudinal motion during acceleration by a step voltage with droop has been theoretically analysed and examined by multi-particle simulations, assuming the machine and beam parameters in the KEK-PS, in which the POP experiment of the induction synchrotron is scheduled. It has turned out that the droop voltage seriously affects on the super-bunch shape from the beginning of acceleration, and even induces particle trapping into the double-well potential originating from the droop. In the case of a synchrotron with a transition energy at the mid-stage of acceleration, mismatching between the largely modified bunch shape before the transition and the bucket shape after the transition leads to an extremely large emittance blow-up. This fact strongly suggests the importance of a droop correction for the acceleration of a super-bunch in an induction synchrotron, where particles are accelerated through the transition energy. A technique to superimpose several pulse voltages of half-sine in time, which can generate a required linearly ramping voltage, has been demonstrated in Ref.[7, 8]. On the other hand, its notable effects have not been observed in the case of an RF-bunch. This means that a droop correction is unnecessary for the first step of the POP experiment at the KEK-PS, in which a single RF bunch will be acceleration by a 250 nsec long induction step voltage.

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